ICCB Adult Education & Family Literacy

Illinois ABE/ASE Curriculum Guide for Mathematics

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Foreword

What are Content Standards?

Content standards describe what students should know and be able to do in a specific content area. The Illinois ABE/ASE Content Standards broadly define what students who are studying reading, writing, and math should know and be able to do as a result of ABE/ASE instruction at a particular level. Content standards also help teachers ensure their students have the skills and knowledge they need to be successful by providing clear goals for student learning.

The Illinois ABE/ASE Content Standards should be used as a basis for curriculum design and may also be used to assist programs and teachers with selecting or designing appropriate instructional materials, instructional techniques, and ongoing assessment strategies. Standards do not tell teachers how to teach, but they do help teachers figure out the knowledge and skills their students should have so that teachers can build the best lessons and environments for their classrooms.

Design of the Illinois ABE/ASE Content Standards

Adult education programs nationwide use the NRS educational functioning levels to provide information to the federal government about student progress. This uniform implementation makes it possible to compare data across programs. The Illinois ABE/ASE content standards conform to the NRS structure for consistency and accountability. There are six NRS educational functioning levels from beginning literacy and adult basic education through adult secondary education. The six levels each have titles and are identified by grade equivalency:

	NRS Educational Functioning Levels	*Grade Level Equivalency*
1	Beginning ABE Literacy	0 - 1.9
2	Beginning Basic Education	2.0 - 3.9
3	Low Intermediate Basic Education	4.0 - 5.9
4	High Intermediate Basic Education	6.0 - 8.9
5	Low Adult Secondary Education	9.0 - 10.9
6	High Adult Secondary Education	11.0 - 12.9

What is Curriculum? How does it differ from Content Standards?

Content standards identify the learning outcomes students should demonstrate at each NRS level. They describe what students should know and be able to do in a specific content area at a specific achievement level.

Curriculum identifies what students need to understand in order to acquire the skills and competencies of each content standard. To develop this curriculum guide, each content standard was examined in light of two key questions:

- What are the essential understandings or skills a student must have in order to obtain the standard?

- What are the essential questions a student or teacher might ask that would move skill acquisition forward?

Through answering these two key questions, programs and teachers decide what they should teach, how they will teach it, and what students will do to demonstrate mastery. Course outlines are developed, materials selected or created, methodologies and assessments designed, and lesson plans built. This becomes the curriculum.

How should the Curriculum Guide for Mathematics be used?

The curriculum guide is meant to be as a companion piece with the Illinois ABE/ASE Mathematics Content Standards as programs and teachers are developing curriculum and planning instruction. The content standards provide greater detail in delineating the exact skills and understandings students should develop in each area and at each level. Teachers and programs should use both documents together in order to have maximum information about each content area. This curriculum guide supports what ABE/ASE students should understand and be able to do in their study of language arts. It is designed to help instructors center learning around essential understandings and essential questions.

The Illinois ABE/ASE Mathematics Content Standards can be found at:
www.iccb.state.il.us/adulted.html

Assessment

Ongoing assessment should be a part of every lesson. Students can demonstrate their mastery of a particular standard through ongoing assessment strategies such as demonstrations, project-based learning, presentations, simulation, out-of-class activities, and other nontraditional assessment strategies. Ongoing assessment is an integral part of instruction in standards-based education.

How to Read the Curriculum Guide for Mathematics

This curriculum guide is organized around NRS levels and content areas in the same manner as the contents standards. Essential understandings and essential questions have been written for each sub-domain area.

Domains are the entire group of related standards in a specific NRS level. Standards from different domains may sometimes be closely related.

Sub-domains are a group of standards within a domain that have separate conceptual categories relative to other sub-domains.

Standards define what students should understand and be able to do.

Domain

NUMBER AND QUANTITY (N)
The Real Number System (RN)
5.N.RN.1 / 5.N.RN.2 / 5.N.RN.3
Essential Understandings: • Rational expressions can be written equivalently using rational exponents. • Properties of integer exponents may be applied to expressions with rational exponents.
Essential Questions: • How can radical and rational exponents be written equivalently? • How do the properties of integer exponents apply to rational exponents?
Quantities (Q)
5.NQ.1
Essential Understandings: • Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations. • Arithmetic and algebra can be used together, with the rules of conversion to transform units.
Essential Questions: • When is it advantageous to represent relationships between quantities symbolically? Numerically? Graphically? • How can the units used in a problem help determine a solution strategy?

Standards → 5.N.RN.1 / 5.N.RN.2 / 5.N.RN.3

Sub-Domain → The Real Number System (RN); Quantities (Q)

Standard → 5.NQ.1

The first number of each standard refers to its NRS level. The letters indicate the domain and sub-domain (if applicable) by code; and the final number indicates the number of the skill within each domain/sub-domain. For example, "5.N.RN.1" labels this standard at NRS Level 5, Math standard #1 in the domain of Number and Quantity with the sub-domain of the Real Number System. See the Introduction to the Mathematics Standards in the Illinois ABE/ASE Mathematics Content Standards for more information on how to read the six NRS levels.

This document does not dictate curriculum or teaching methods. What adult students can learn at any particular level depends upon what they have learned before. Learning opportunities will continue to vary across the state of Illinois adult education programs, and educators should make every effort to meet the needs of all students based on their current understanding.

Understanding the Mathematical Curriculum Guide[1]

This curriculum guide supports what ABE/ASE adult learners should understand and be able to do in their study of mathematics. It is designed to help instructors center learning around essential understandings and essential questions. Assessing whether a student understands a mathematical concept involves their display of higher order thinking skills. One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The standards are level specific but do not define the intervention methods or materials necessary to support adult learners who are well below or well above level expectations. It is also beyond the scope of this curriculum guide to define the full range of supports appropriate for English language learners (ELL) and for students with special needs and abilities. At the same time, all students need to have the opportunity to learn and meet the same high standards necessary to succeed and transition to college and/or careers.

[1] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

ELL students have the ability to participate in mathematical discussions as they learn English. Mathematics instruction for ELL students can draw on multiple resources and modes available in the classroom---such as objects, drawings, inscriptions, and gestures---as well as mathematical experiences outside of the classroom.

Promoting a culture of high expectations for all students is a fundamental goal of adult education. In order to participate with success, both ELL and students with disabilities should be provided the necessary supports, accommodations, and services where applicable.

No set of level-specific standards or curriculum can fully reflect the great variety in the abilities, needs, learning rates, and achievement levels of adult learners in any given classroom. However, the Illinois ABE/ASE Content Standards along with this curriculum guide do provide clear signposts along the way to the goal of college and career readiness for all students.

Mathematical Practices

The Illinois ABE/ASE Standards for Mathematical Practice describe a variety of expertise that mathematics adult educators at all levels should seek to develop in their adult students[2]. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

<u>1. Make sense of problems and persevere in solving them.</u>
Mathematically proficient adult learners start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a

[2] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Higher level students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient adult learners can explain correspondences between equations, verbal descriptions, tables, and graphs, or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Lower level students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient adult learners make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize (to abstract a given situation, represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents); and the ability to contextualize (to pause as needed during the manipulation process in order to probe into the referents for the symbols involved). Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient adult learners understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient adult learners are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Lower level adult learners can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until higher levels. Later, students learn to determine domains to which an argument applies. Students at all levels can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient adult learners can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In lower levels, this might be as simple as writing an addition equation to describe a situation. In intermediate levels, adult learners might apply proportional reasoning to plan a school event or analyze a problem in the community. By the upper levels, an adult learner might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient adult learners who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient adult learners consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient adult students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient advanced level adult learners analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient adult learners try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the lower levels, adult learners give carefully formulated explanations to each other. By the time they reach the advanced levels, they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient adult learners look closely to discern a pattern or structure. Lower level students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, higher level students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient adult learners notice if calculations are repeated and look both for general methods and for shortcuts. Intermediate level students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, intermediate level students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient adult students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Curriculum Guide to Mathematical Practices[3]

The Mathematical Practices describe ways in which adult learners should increasingly engage with the subject matter as they grow in mathematical maturity and expertise. All adult educators, instructional leaders, designers of curricula, assessments, and professional development should connect the curriculum guide to mathematical practices.

This curriculum guide is a combination essential understandings and questions. Essential Understandings are exceptional opportunities to connect the practices to mathematical content. Adult learners who lack understanding of a topic may rely on procedures too heavily. Without substantial mathematical understanding from which to work, adult learners may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents an adult learner from engaging in the mathematical practices. In this respect, these essential understandings are potential “points of intersection” between this guide and the Mathematical Practices.

[3] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

Curriculum Guide for Mathematics

NRS Levels 1-6

NRS Level 1 Overview

Counting and Cardinality / Numeracy (CC)

- Know number names and the count sequence.
- Count to tell the number of objects
- Compare numbers

Operations and Algebraic Thinking (OA)

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
- Represent and solve problems involving addition and subtraction
- Understand and apply properties and the relationship between addition and subtraction
- Add and subtract within 20
- Work with addition and subtraction equations

Number and Operations in Base Ten (NBT)

- Extend the counting sequence
- Work with numbers 11-19 and tens to gain foundations for place value
- Use place value understanding and properties of operations to add and subtract

Measurement and Data (MD)

- Describe and compare measurable attributes
- Classify objects and count the number of objects in each category
- Measure lengths indirectly and by iterating length units
- Tell and write time
- Represent and interpret data

Geometry (G)

- Identify and describe shapes
- Analyze, compare, create, and compose shapes
- Reason with shapes and their attributes

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

OVERVIEW EXPLANATION OF MATHEMATICS
NRS Level 1 – Beginning ABE Literacy
(Grade Levels 0 – 1.9)

At the Beginning ABE Literacy Level, instructional time should focus on four critical areas[4]:

1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;
2. Developing understanding of whole number relationships and place value, including grouping in tens and ones;
3. Developing understanding of linear measurement and measuring lengths as iterating length units; and
4. Describing, reasoning about attributes of, composing and decomposing geometric shapes.

This Curriculum Guide complements the Illinois ABE/ASE Mathematical Content Standards. More information on the above can be found there.

[4] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

NRS Level 1 – Beginning ABE Literacy
(Grade Levels 0 – 1.9)

COUNTING AND CARDINALITY / NUMERACY (CC)
1.CC.1 / 1.CC.2 / 1.CC.3 / 1.CC.4 / 1.CC.5 / 1.CC.6 / 1.CC.7
Essential Understandings: • Counting determines how many or how much a quantity/number represents. • When counting, the last number spoken is the total number of objects. • Counting one more will be the next larger number. • Each successive number name refers to a quantity that is one larger. • Knowledge of numbers 0-10 can be applied to predict order and sequence in higher numbers (10-20, 20-30, etc.) • Quantities of numbers can be compared, ordered, and described as less than, greater than, or equal to one another. • A written number represents an amount/quantity/order and each number represents a different amount/quantity/order.
Essential Questions: • Why/when are objects counted? What objects are/can be counted? • How is number order helpful to us? • What can numerals represent? • How would you describe a teen number? • How can you use 0-10 to predict other counting sequences?

OPERATIONS AND ALGEBRAIC THINKING (OA)
1.OA.1 / 1.OA.2 / 1.OA.3 /1.OA.4 / 1.OA.5 / 1.OA.6 / 1.OA.7 / 1.OA.8 / 1.OA.9 / 1.OA.10 / 1.OA.11 / 1.OA.12 / 1.OA.13

Essential Understandings:

- Addition and subtraction can be represented by objects, drawings, manipulatives, and other modalities.
- Expressions and equations can be used to decompose numbers in more than one way.
- Quantities can be used to create a variety of individual groupings.
- Numbers less than or equal to 20 can be decomposed by adding, subtracting, or re-grouping.
- The whole is equal to the sum of its parts; conversely, the whole minus a part is equal to the other part.
- Strategies (for example, properties of addition) can be used to decompose complex problems to make an easier problem (counting on, make a ten, near ten, doubles, plus one, plus two, etc.)
- Problem solving structures reinforce part/whole and number combinations within 20
- Word problems have basic problem solving structures: adding to, taking from, putting together, taking apart, comparing and can be represented using different modalities.
- Unknowns can be in various locations (start, change, result) in equations and develop from combinations of numbers.
- Addition and subtraction are related/inverse operations.
- Various strategies can be used to quickly add numbers.
- The equal sign is used to represent quantities that have the same value.

Essential Questions:

- Why should numbers be decomposed to form different combinations of a specific number?
- What is the connection of a number to an equation or expression?
- How are word problems connected to an equation or expression?
- Why is it important to know multiple strategies in solving addition/subtraction problems?
- How are problem solving strategies and/or properties connected to number relationships?
- What is the relationship between addition and subtraction?
- How can word problems be decoded into equations or expressions to solve it?
- Does a solution make the equation true or false? How is a solution evaluated and does it make sense?

NUMBER AND OPERATIONS IN BASE TEN (NBT)

1.NBT.1 / 1.NBT.2 / 1.NBT.3 / 1.NBT.4 / 1.NBT.5 / 1.NBT.6

Essential Understandings:

- Two digit numbers are composed of groups of tens and ones and can be compared with symbols (<, >, =) in terms of their relationship.
- Various models can be used to build individual numbers with tens/ones while counting.
- Counting sequences can be used to understand counting by 10s, identifying 10 more, 10 less.
- Counting can be connected to adding and subtracting.
- Addition can be used to solve and/or evaluate subtraction and vice versa.
- Mental math can be used to check and/or perform calculations in base 10.

Essential Questions:

- How do addition and subtraction relate to counting sequences?
- How does understanding properties of operations help with strategies when performing written and mental calculations?
- How does using objects and drawings help represent problems in multiple ways?
- What is significant about 10?
- What is significant about the teen numbers and how do these numbers relate to 10? (e.g., 10 + 3 = 13).

MEASUREMENT AND DATA (MD)

1.MD.1 / 1.MD.2 / 1.MD.3 / 1.MD.4 / 1.MD.5 / 1.MD.6 / 1.MD.7

Essential Understandings:

- Some attributes are measurable; both numbers and words can be used to describe and compare the measurements.
- Objects can be classified, ordered, and compared by attributes and/or measurement.
- Time is measured in hours and half-hours using analog and digital clocks.
- Data can be organized and classified by comparing attributes (height, width and depth).

Essential Questions:

- How are measurable attributes determined and why are these attributes of objects important to comparing quantities?
- How are dividing a circle and telling time related?
- What is the purpose of categorizing data?
- What strategies can be used to organize data?

GEOMETRY (G)
1.G.1 / 1.G.2 / 1.G.3 / 1.G.4 / 1.G.5 / 1.G.6 / 1.G.7 / 1.G.8 / 1.G.9
Essential Understandings: • Objects have position relative to other objects using terms such as "above," "below," "beside," "in front of," "behind," and "next to." • Two-dimensional shapes are flat and can be built from components. • Three-dimensional shapes have unique attributes and specific names regardless of their orientations or overall size. • Attributes are used to compare and analyze two- and three-dimensional shapes. • Circles and rectangles can be used to create more complex shapes; circles and rectangles can be partitioned into equal shares. • Shapes can be used to build pictures, designs and other shapes. • Understanding of shapes and components to recognize and represent shapes in the world.
Essential Questions: • Why are positional words important in math? • How can shapes be partitioned into halves and quarters? • Why is mathematical language critical when describing two-dimensional and three-dimensional shapes? • How can two-dimensional shapes be decomposed or combined to form two- or three-dimensional shapes and vice versa?

NRS Level 2 Overview

Operations and Algebraic Thinking (OA)

- Represent and solve problems involving addition and subtraction
- Add and subtract within 20
- Work with equal groups of objects to gain foundations for multiplication
- Represent and solve problems involving multiplication and division
- Understand properties of multiplication and the relationship between multiplication and division
- Multiply and divide within 100
- Solve problems involving the four operations and identify and explain patterns in arithmetic

Number and Operations in Base Ten (NBT)

- Understand place value
- Use place value understanding and properties of operations to add and subtract and to perform multi-digit arithmetic

Number and Operations – Fractions (NF)

- Develop understanding of fractions as numbers

Measurement and Data (MD)

- Measure and estimate lengths in standard units
- Relate addition and subtraction to length
- Work with time and money
- Represent and interpret data
- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition
- Geometric measurement: recognize perimeter as an attribute of plan figures an distinguish between linear and area measures

Geometry (G)

- Reason with shapes and their attributes

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity i repeated reasoning.

OVERVIEW EXPLANATION OF MATHEMATICS
NRS Level 2 – Beginning Basic Education
(Grade Levels 2.0 – 3.9)

At the Beginning Basic Education Level, instructional time should focus on six critical areas[5]:

1. Extending understanding of base-ten notation;
2. Building fluency with addition and subtraction;
3. Developing understanding of multiplication and division and strategies for multiplication and division within 100;
4. Developing understanding of fractions, especially unit fractions (fractions with numerator 1);
5. Using standard units of measure and developing understanding of the structure of rectangular arrays and of area; and
6. Describing and analyzing one- and two-dimensional shapes.

This Curriculum Guide complements the Illinois ABE/ASE Mathematical Content Standards. More information on the above can be found there.

[5] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

NRS Level 2 – Beginning Basic Education
(Grade Levels 2.0 – 3.9)

OPERATIONS AND ALGEBRAIC THINKING (OA)
2.OA.1 / 2.OA.2 / 2.OA.3 / 2.OA.4 / 2.OA.5 / 2.OA.6 / 2.OA.7 / 2.OA.8 / 2.OA.9 / 2.OA.10 / 2.OA.11 / 2.OA.12 / 2.OA.13
Essential Understandings: • There are different problem solving structures that can be used to solve problems in multiple ways. • Unknown quantities can be represented in different places in an equation/number model. • Addition and subtraction can be represented on various models such as number lines, picture graphs, algebra tiles, and bar graphs. • Word problems can be structured to require multi-step solutions. • Fluency with all sums, differences, products, and quotients of two numbers (0-12). • Even numbered objects can be modeled using pairs or rectangular arrays. • The difference between even and odd numbers. • Visual images and numerical patterns of multiplication and division can be used in problem-solving situations. • The Properties of Operations will help in performing computations as well as in problem-solving situations (Distributive, Associative, Commutative, Identity, and Zero.)
Essential Questions: • How does an equation represent an unknown quantity? • How do visual representations depict and help solve addition, subtraction, multiplication, and division problems? • How does fluency with basic sums, differences, products, and quotients help in problem solving situations? • What are efficient methods for finding sums and differences using even and odd properties of numbers? • How do multiples and factors relate to multiplication and division? • How can inverse operations be used to solve problems? • How can the reasonableness of a solution be evaluated? • How can arithmetic patterns be used to help find solutions to problems? • What are some of the rules or properties of whole numbers?

NUMBERS AND OPERATIONS IN BASE TEN (NBT)
2.NBT.1 / 2.NBT.2 / 2.NBT.3 / 2.NBT.4 / 2.NBT.5 / 2.NBT.6 / 2.NBT.7 / 2.NBT.8 / 2.NBT.9 / 2.NBT.10 / 2.NBT.11
Essential Understandings: • Numbers are composed of other numbers. • Numbers can represent quantity, position, location and relationships. • Place value is based on groups of ten. • Flexible methods of computation involve grouping numbers in strategic ways. • There are different problem solving structures that can be used to solve problems in multiple ways. • Strategies based on place value and properties of operations can be used to represent the product of one digit whole numbers by multiples of 10 (in the range of 10-90).
Essential Questions: • How can numbers be expressed, ordered and compared? • How does the position of a digit in a number affect its value? • In what ways can numbers be composed and decomposed using addition, subtraction and multiplication? • What are efficient methods for finding sums and differences?
NUMBERS AND OPERATIONS IN FRACTIONS (NF)
2.NF.1 / 2.NF.2 / 2.NF.3
Essential Understandings: • The size of the fractional part is relative to the size of the whole. • Fractions are quantities where a whole is divided into equal-sized parts and can be represented by models (such as, rulers, manipulatives, words, and/or number lines, etc.) • Fractions can be used as a tool to understand and model quantities and relationships. • Fractions are composed of unit fractions. • Fractions that represent equal-sized quantities are equivalent. • Two fractions with the same numerator represent the same number of parts. • Two fractions with the same denominator represent the same number of parts of the whole. • Whole numbers can be represented as a fraction such as $3 = \frac{3}{1}$ or any fraction whose numerator and denominator are the same is equal to 1, such as $\frac{4}{4} = 1$.
Essential Questions: • What do fractions represent? • What makes fractions equivalent? • What is the relationship between two fractions with the same numerator or two fractions with the same denominator?

MEASUREMENT AND DATA (MD)
2.MD.1 / 2.MD.2 / 2.MD.3 / 2.MD.4 / 2.MD.5 / 2.MD.6 / 2.MD.7 / 2.MD.8 / 2.MD.9 / 2.MD.10 / 2.MD.11 / 2.MD.12 / 2.MD.13 / 2.MD.14 / 2.MD.15 / 2.MD.16
Essential Understandings: • There is a relationship between estimation and measurement. • Measurement is a way to describe and compare objects or ideas. • A specific process or tool (i.e., a metric or standard ruler) can be used to measure attributes of unit length. • Metric measurement units are related to place value concepts/multiples of 10. • A number line is used to represent measurement attributes such as, distance and quantity. • Currency has different values and is counted according to its values. • Standard units provide common language for communicating time. • Equivalent periods of units are used to measure time. • Information can be represented in scaled bar and picture graph form. These graphs can be used to help solve one and two- step math problems. • Elapsed time is the interval of time, given a specific unit, from a starting time to an ending time. • Perimeter and addition are related. • A linear unit is used to measure perimeter. • Everyday objects have a variety of attributes, each of which can be measured in many ways. • Area can be a function of addition as well as multiplication. • Perimeter and area are related. • Modeling (tiling) multiplication and decomposing problems based upon their problem-solving structure can help in finding solutions. • The mass (two-dimensional figures) and volume (three-dimensional figures) of a substance or solid can be measured and expressed in terms of standard units (square or cubic units).

Essential Questions:

- When is it appropriate to estimate and when is it appropriate to provide an exact answer?
- What properties or attributes can be measured?
- How are attributes measured (unit, tool, and process)?
- How can accurate measurements solve problems and make sense of the world?
- How does monetary value affect how money is counted?
- How do units within a system relate to each another?
- How are various representations of time related?
- How can understanding the relationship between addition and subtraction aid in problem solving?
- How can data represented in scaled bar and picture graphs be useful in the real world?
- What conclusions can be made about elapsed time and its usefulness?
- How can understanding the relationship between addition and area aid in problem solving?
- How can modeling multiplication and decomposing problems help in finding their solutions?
- What is the relationship between area and addition/multiplication?
- How does metric measurement connect to multiples of 10?
- How does volume or mass of a three-dimensional figure differ from the area of a two-dimensional figure? (Describe in terms of units and/or attributes of each figure.)

GEOMETRY (G)

2.G.1 / 2.G.2 / 2.G.3 / 2.G.4 / 2.G.5

Essential Understandings:

- Any geometric figure can be composed or decomposed from/into other figures whose areas are the sum of its parts.
- Some objects can be described and compared using their geometric attributes (which may be fractional components).

Essential Questions:

- How can the attributes of any geometric figure be composed or decomposed to represent or model the sum of its parts?
- What is the significance of composing or decomposing a geometric figure into the sum of its parts?
- How can plane (two-dimensional) and solid (three-dimensional) shapes be described?

NRS Level 3 Overview

Operations and Algebraic Thinking (OA)

- Use the four operations with whole numbers to solve problems
- Gain familiarity with factors and multiples
- Generate and analyze patterns
- Write and interpret numerical expressions
- Analyze patterns and relationships

Number and Operations in Base Ten (NBT)

- Generalize place value understanding for multi-digit whole numbers
- Use place value understanding and properties of operations to perform multi-digit arithmetic
- Understand the place value system
- Perform operations with multi-digit whole numbers and with decimals to hundredths

Number and Operations – Fractions (NF)

- Extend understanding of fraction equivalence and ordering
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers
- Understand decimal notation for fractions, and compare decimal fractions
- Use equivalent fractions as a strategy to add and subtract fractions
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions

Measurement & Data (MD)

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit
- Represent and interpret data
- Geometric measurement: understand concepts of angle and measure angles
- Convert like measurement units within a given measurement system
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition

NRS Level 3 Overview (continued)

Geometry (G)

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles
- Graph points on the coordinate plane to solve real-world and mathematical problems
- Classify two-dimensional figures into categories based on their properties

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

OVERVIEW EXPLANATION OF MATHEMATICS
NRS Level 3 – Low Intermediate Basic Education
(Grade Levels 4.0 – 5.9)

At the Low Intermediate Basic Education Level, instructional time should focus on six critical areas[6]:

1. Developing understanding and fluency with multi-digit multiplication, and of dividing to find quotients involving multi-digit dividends;
2. Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers;
3. Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions);
4. Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations;
5. Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry; and
6. Developing understanding of volume.

This Curriculum Guide complements the Illinois ABE/ASE Mathematical Content Standards. More information on the above can be found there.

[6] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

NRS Level 3 – Low Intermediate Basic Education
(Grade Levels 4.0 – 5.9)

OPERATIONS AND ALGEBRAIC THINKING (OA)
3.OA.1 / 3.OA.2 / 3.OA.3 / 3.OA.4 / 3.OA.5 / 3.OA.6 / 3.OA.7 / 3.OA.8
Essential Understandings: • Flexible methods of computation involve grouping numbers in strategic ways. • The distributive property is connected to the area model and/or partial products method of multiplication. • Some division situations will produce a remainder, but the remainder should always be less than the divisor. If the remainder is greater than the divisor, that means at least one more can be given to each group (fair sharing) or at least one more group of the given size (the dividend) may be created. When using division to solve word problems, how the remainder is interpreted depends on the problem situation. • Number or shape patterns are generated by following a given rule. • The four operations (addition, subtraction, multiplication, and division) are interconnected. • Parentheses, brackets, and braces are used to guide the order of operations when simplifying expressions. • A standard algorithm is used to fluently multiply multi-digit whole numbers. • A variety of different strategies can be used to multiply and divide multi-digit numbers including: visual models (rectangular array, equations, and/or area model). • Strategies for multiplication and division are based on place value, the properties of operations, and/or the relationship between multiplication and division (approaching problems with unknown product of quotient, group size unknown, and number of groups unknown).

Essential Questions:

- How do I determine the factors of a number?
- What is the difference between a prime and composite number?
- How are multiplication and division related to each other?
- What are efficient methods for finding products and quotients, and how can place value properties aid computation?
- How are dividends, divisors, quotients, and remainders related?
- How are the four operations of addition, subtraction, multiplication and division used in multi-step word problems? (How can these operations be used to assess the reasonableness of a solution?)
- How can a remainder be interpreted with respect to the answer in a division word problem? (Is the solution reasonable?)
- How do parentheses, brackets, and braces affect the way expressions are simplified or evaluated?
- When are different strategies appropriate to use when multiplying and/or dividing multi-digit numbers?
- What strategies can be used to find rules for patterns and what predictions can the pattern support?

NUMBER AND OPERATIONS IN BASE TEN (NBT)

3.NBT.1 / 3.NBT.2 / 3.NBT.3 / 3.NBT.4 / 3.NBT.5 / 3.NBT.6 / 3.NBT.7 / 3.NBT.8 / 3.NBT.9 / 3.NBT.10 / 3.NBT.11 / 3.NBT.12 / 3.NBT.13 / 3.NBT.14 / 3.NBT.15

Essential Understandings:

- The place value of whole and decimal numbers is based on groups of ten and the value of a number is determined by the place of its digits.
- The standard algorithm for addition and subtraction relies on adding or subtracting like base-ten units.
- Whole numbers are read from left to right using the name of the period; commas are used to separate periods.
- A whole or decimal number can be written using its name, standard, or expanded form and can be compared to other whole or decimal numbers using greater than, less than or equal to symbols.
- Flexible methods of computation involve grouping numbers in strategic ways.
- Multiplication and division are inverse operations.
- The four operations are interconnected.
- In a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.
- Multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. The exponent not only indicates how many places the decimal point is moving but also that you are multiplying or making the number 10 times greater, three times when you multiply by 10^3 (e.g. $3.4 \times 10^3 = 3.4 \times (10 \times 10 \times 10) = 3.4 \times 1{,}000 = 3{,}400$).

Essential Questions:

- How does the position of a digit in a number affect its value, and how can the value of digits be used to compare two numbers?
- In what ways can numbers be composed and decomposed?
- How are the four basic operations related to one another?
- How does understanding place value help you solve multi-digit addition and subtraction problems and how can rounding be used to estimate answers to problems?
- What occurs when whole numbers and decimals are multiplied by 10 or powers of 10?
- Using less than, greater than, or equal to symbols, how can whole and decimal numbers (with like or unlike forms) be compared?

NUMBER AND OPERATIONS - FRACTIONS (NF)

3.NF.1 / 3.NF.2 / 3.NF.3 / 3.NF.4 / 3.NF.5 / 3.NF.6 / 3.NF.7 / 3.NF.8 / 3.NF.9 / 3.NF.10 / 3.NF.11 / 3.NF.12 / 3.NF13 / 3.NF.14

Essential Understandings:

- Fractions can be represented visually and in written form.
- Comparisons are valid only when the fractions or decimal numbers refer to the same whole.
- Fractions and mixed numbers are composed of unit fractions and can be decomposed as a sum of unit fractions.
- Improper fractions and mixed numbers can represent the same value.
- Addition and subtraction of fractions involves joining and separating parts referring to the same whole.
- A product of a fraction times a whole number can be written as a multiple of a unit fraction.
- Fractions with denominators of 10 can be expressed as an equivalent fraction with a denominator of 100.
- Fractions with denominators of 10 and 100 may be expressed using decimal notation.
- Benchmark fractions and other strategies aid in estimating the reasonableness of results of operations with fractions.
- The use of area models, fraction strips, and number lines, are effective strategies to model sums, differences, products, and quotients.
- Equivalent fractions are critical when adding and subtracting fractions with unlike denominators.
- Fractions are division models.
- Multiplication can be interpreted as scaling/resizing (multiplying a given number by a fraction greater than 1 results in a product greater than the given number and multiplying a given number by a fraction less than 1 results in a product smaller than the given number).
- The knowledge of fractions and equivalence of fractions can be used to develop algorithms for adding, subtracting, multiplying, and dividing fractions.

Essential Questions:

- How are fractions used in problem-solving situations?
- How are fractions composed, decomposed, compared and represented?
- Why is it important to identify, label, and compare fractions as representations of equal parts of a whole or of a set?
- How can multiplying a whole number by a fraction be displayed as repeated addition (as a multiple of a unit fraction)?
- How can visual models be used to determine and compare equivalent fractions and decimals?
- How can decimals through the hundredths place be compared and ordered?
- What is a reasonable estimate for a solution (answers)?
- How do operations with fractions relate to operations with whole numbers?
- What do equivalent fractions represent and why are they useful when solving equations with fractions?
- What models or pictures could aid in understanding a mathematical or real-world problem and the relationships among the quantities?
- When can model(s) or picture(s) be used to solve a mathematical or real-world problem?
- What are the effects of multiplying by quantities greater than one compared to the effects of multiplying by quantities less than one?

MEASUREMENT AND DATA (MD)

3.MD.1 / 3.MD.2 / 3.MD.3 / 3.MD.4 / 3.MD.5 / 3.MD.6 / 3.MD.7 / 3.MD.8 / 3.MD.9 / 3.MD.10 / 3.MD.11 / 3.MD.12

Essential Understandings:

- Converting from larger to smaller units of measurement in the metric system is done by multiplying by powers of ten.
- Perimeter is a real life application of addition and subtraction.
- Area is a real life application of multiplication and division.
- When converting measurements within one system, the size, length, mass, volume of the object remains the same.
- Measurement problems can be solved by using appropriate tools.
- Volume of three-dimensional figures is measured in cubic units.
- Volume is additive and/or it is the multiplication of three dimensions (length, width and height).
- Multiple rectangular prisms can have the same volume.
- Volume can be used to solve a variety of real life problems.
- The concepts of distances, intervals of time, volume, masses of objects, and money can be expressed as measurements of a larger unit in terms of a smaller unit.
- Angles are measured in the context of a central angle of a circle.
- Angles are composed of smaller angles.

Essential Questions: • How are the units of measure within the metric system related? • How do you find the area and perimeter of geometric figures and how can using the formulas for perimeter and area help you solve real-world problems? • Why does the size, length, mass, volume of an object remain the same when converted to another unit of measurement? • What is volume and how is it used in real life? • How does the area of rectangles relate to the volume of rectangular prisms? • What are the types of angles and the relationships? • How are angles applied in the context of a circle? • How are protractors used to measure and aid in drawing angles and triangles? • How can an addition or subtraction equation be used to solve a missing angle measure when the whole angle has been divided into two angles and only one measurement is given?
GEOMETRY (G)
3.G.1 / 3.G.2 / 3.G.3 / 3.G.4 / 3.G.5 / 3.G.6 / 3.G.7
Essential Understandings: • Shapes can be classified by properties (or attributes) of their lines and angles. • Angles are measured in the context of a central angle of a circle. • Angles are composed of smaller angles. • Two-dimensional geometric figures are composed of various parts that are described with precise vocabulary and can be classified based upon their properties (attributes). • In a coordinate plane, the first number indicates how far to travel from the origin in the direction of one axis and the second number indicates how far to travel in the direction of the second axis. • The coordinate plane can be used to model and compare numerical patterns. • Figures that can be folded on a center line to produce two matching parts are symmetrical.

Essential Questions:

- How are parallel lines and perpendicular lines used in classifying two-dimensional shapes?
- What are the types of angles and the relationships?
- How are angles applied in the context of a circle?
- How are protractors used to measure and aid in drawing angles and triangles?
- Why is it important to use precise language and mathematical tools in the study of two-dimensional figures?
- How can describing, classifying and comparing properties of two-dimensional shapes be useful in solving real-world problems?
- How can an addition or subtraction equation be used to solve a missing angle measure when the whole angle has been divided into two angles and only one measurement is given?
- What is the purpose of a coordinate plane?
- How can graphing points on the coordinate plane help to solve real world and mathematical problems?
- How can the line of symmetry be identified and drawn in a two-dimensional figure?

NRS Level 4 Overview

Ratios and Proportional Relationships (RP)

- Understand ratio concepts and use ratio reasoning to solve problems
- Analyze proportional relationships and use them to solve real-world and mathematical problems

The Number System (NS)

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions
- Compute fluently with multi-digit numbers and find common factors and multiples
- Apply and extend previous understandings of numbers to the system of rational numbers
- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers
- Know that there are numbers that are not rational, and approximate them by rational numbers

Expressions and Equations (EE)

- Apply and extend previous understandings of arithmetic to algebraic expressions
- Reason about and solve one-variable equations and inequalities
- Represent and analyze quantitative relationships between dependent and independent variables
- Use properties of operations to generate equivalent expressions
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations
- Work with radicals and integer exponents
- Understand the connections between proportional relationships, lines, and linear equations
- Analyze and solve linear equations and pairs of simultaneous linear equations

Functions (F)

- Define, evaluate, and compare functions
- Use functions to model relationships between quantities

NRS Level 4 Overview (continued)

Geometry (G)

- Solve real-world and mathematical problems involving area, surface area, and volume
- Draw, construct, and describe geometrical figures and describe the relationship between them
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume
- Understand congruence and similarity using physical models, transparencies, or geometry software
- Understand and apply the Pythagorean Theorem
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres

Statistics and Probability (SP)

- Develop understanding of statistical variability
- Summarize and describe distributions
- Use random sampling to draw inferences about a population
- Draw informal comparative inferences about two populations
- Investigate chance processes and develop, use, and evaluate probability models
- Investigate patterns of association in bivariate data

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

OVERVIEW EXPLANATION OF MATHEMATICS
NRS Level 4 – High Intermediate Basic Education
(Grade Levels 6.0 – 8.9)

At the High Intermediate Basic Education Level, instructional time should focus on seven critical areas[7]:

1. Developing understanding of and applying proportional relationships.
2. Developing understanding of operations with rational numbers and working with expressions and linear equations.
3. Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations.
4. Grasping the concept of a function and using functions to describe quantitative relationships.
5. Solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume.
6. Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.
7. Drawing inferences about populations based on samples, they build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations.

This Curriculum Guide complements the Illinois ABE/ASE Mathematical Content Standards. More information on the above can be found there.

[7] Common Core State Standards for Mathematics, 2010, www.corestandards.org.

NRS Level 4 – High Intermediate Basic Education
(Grade Levels 6.0-8.9)

RATIOS AND PROPORTIONAL RELATIONSHIPS (RP)
4.RP.1 / 4.RP.2 / 4.RP.3 / 4.RP.4 / 4.RP.5 / 4.RP.6
Essential Understandings: • A ratio expresses the comparison between two quantities. Special types of ratios are rates, unit rates, measurement conversions, and percent. • A ratio or a rate expresses the relationship between two quantities. Ratio and rate language are used to describe a relationship between two quantities (including unit rates). • A rate is a type of ratio that represents a measure, quantity, or frequency, typically one measured against a different type of measure, quantity, or frequency. • Ratio and rate reasoning can be applied to many different types of mathematical and real-life problems (rate and unit rate problems, scaling, unit pricing, statistical analysis, etc.). • Rates, ratios, percentages and proportional relationships express how quantities change in relationship to each other and can be represented in multiple ways. • Rates, ratios, percentages and proportional relationships can be applied to multi-step ratio and percent problems along with other problem solving situations such as interest, tax, discount, etc.
Essential Questions: • When is it useful to be able to relate one quantity to another? • How are ratios and rates similar and different? • What is the connection between a ratio/rate and a fraction? • How do rates, ratios, percentages and proportional relationships apply to our world? • When and why is it appropriate to use proportional comparisons? • How does comparing quantities describe the relationship between them? • How can models illustrate proportional relationships? • How can proportional relationships be used to solve ratio and percent problems? • How can scale drawings be used to compute actual lengths and area?

THE NUMBER SYSTEM (NS)
4.NS.1 / 4.NS.2 / 4.NS.3 / 4.NS.4 / 4.NS.5 / 4.NS.6 / 4.NS.7 / 4.NS.8 / 4.NS.9 / 4.NS.10 / 4.NS.11 / 4.NS.12 / 4.NS.13

Essential Understandings:

- Rational numbers use the same attributes as whole numbers.
- The quotatative (making groups of a certain size) and partitative (sharing equally or dealing out) types of division and measurement are applied to numbers within the real number system (fractions, decimals, integers and rational and irrational numbers).
- The relationship of the location of the digits and the value of the digits is part of understanding multi-digit operations.
- Various operations can be performed and represented using multiple formats (manipulatives, diagrams, real-life situations, equations).
- Quantities having more or less than zero are described using positive and negative numbers.
- Number lines are visual models used to represent the density principle: between any two whole numbers are many rational numbers, including decimals and fractions.
- The rational numbers can extend to the left or to the right on the number line, with negative numbers going to the left of zero, and positive numbers going to the right of zero.
- The coordinate plane is a tool for modeling real-world and mathematical situations and for solving problems.
- Graphing objects in a four quadrant graph can provide ways to measure distances
- Rational numbers can be represented with visuals (including distance models), language, and real-life contexts.
- There are precise terms and sequence to describe operations with rational numbers.
- Every number has a decimal expansion.
- Properties of operations with whole and rational numbers also apply to all real numbers.
- Absolute value is a number's distance from zero (e.g., $|-3| = 3$).
- The greatest common factor (GCF) and the least common multiple (LCM) among whole numbers can be determined.
- The sum of two whole numbers between 1 and 100 can be expressed as a multiple of a sum of two whole numbers (e.g., the distributive property).

Essential Questions:

- How are various operations (addition, subtraction, multiplication and division) represented, interpreted and related to realistic situations and to other operations?
- What role does place value play in multi-digit operations?
- How are positive and negative numbers used?
- How do rational numbers relate to integers?
- What can be modeled on the coordinate plane?
- What is the relationship between properties of operations and types of numbers?
- Why are quantities represented in multiple ways?
- How can quantities be represented and what is the rationale for selecting a specific representation?
- How is the universal nature of properties applied to real numbers?
- What does the absolute value of a number represent?
- What is the difference between the GCF and LCM?
- How can the distributive property be used to express the sum of two whole numbers [e.g., 25 + 10 as 5(5 + 2)].

EXPRESSIONS AND EQUATIONS (EE)
4.EE.1 / 4.EE.2 / 4.EE.3 / 4.EE.4 / 4.EE.5 / 4.EE.6 / 4.EE.7 / 4.EE.8 / 4.EE.9 / 4.EE.10 / 4.EE.11 / 4.EE.12 / 4.EE.13 / 4.EE.14 / 4.EE.15 / 4.EE.16 / 4.EE.17 / 4.EE.18 / 4.EE.19 / 4.EE.20 / 4.EE.21

Essential Understandings:

- Variables within algebraic expressions are a modeling tool to use when solving real-world problems. This process demonstrates a method of describing quantitative relationships – for instance, traveling some distance (d) at a given rate of travel will take a given amount of time (t) with a constant rate.
- The value of any real number can be represented in relation to other real numbers such as with decimals converted to fractions, scientific notation and numbers written with exponents (e.g., $8 = 2^3$).
- Properties of operations are used to determine if expressions are equivalent.
- Solving equations is a reasoning process and follows established procedures based on properties.
- Substitution is used to determine whether a given number in a set makes an equation or inequality true.
- Variables may be used to represent a specific number, or, in some situations, to represent all numbers in a specified set.
- When one expression has a different value than a related expression, an inequality provides a way to show that relationship between the expressions: the value of one expression is greater than (or greater than or equal to) the value of the other expression instead of being equal.
- Solutions of inequalities can be represented on a number line.
- Variables in algebraic equations can be expressed in graphs to represent numbers and generalize mathematical problems in real-world situations.
- Understand the difference between an expression and an equation: expressions are simplified and equations are solved for the variable's value.
- Properties of operations can be used to add, subtract, factor, and expand linear expressions.
- Expressions can be manipulated to suit a particular purpose to solve problems efficiently.
- Mathematical expressions, equations, inequalities and graphs are used to represent and solve real-world and mathematical problems.
- Properties, order of operations, and inverse operations are used to simplify expressions and solve equations efficiently.
- Unit rates can be explained in graphical representations and algebraic equations.
- The solution to a system of two linear equations in two variables is an ordered pair that satisfies both equations.
- Some equations/inequalities and systems of equations/inequalities have no solutions (parallel lines) and others have infinite solutions (same line).
- Square roots and cube roots of small perfect squares and cubes can be evaluated and/or represent solutions to the equations in the form of $x^2 = y$ and $x^3 = y$ where y is a positive rational number.
- The properties of integer exponents can generate equivalent numerical expressions.

Essential Questions:

- How do we determine if a variable is independent or dependent in an expression or equation?
- What is equivalence?
- How properties of operations used to prove equivalence?
- How are variables defined and used?
- How does the structure of equations and/or inequalities help us solve equations and/or inequalities?
- How does the substitution process help in solving problems?
- Why are variables used in equations?
- What might a variable represent in a given situation?
- How are inequalities represented and solved?
- When and how are expressions, equations, inequalities and graphs applied to real world situations?
- How can the order of operations be applied to evaluating expressions, and solving from one-step to multi-step equations?
- What are some possible real-life situations to which there may be more than one solution?
- How does the ongoing use of fractions and decimals apply to real-life situations?
- How do we express a relationship mathematically?
- How do we determine the value of an unknown quantity?
- What makes a solution strategy both efficient and effective?
- How is it determined if multiple solutions to an equation are valid?
- How does the context of the problem affect the reasonableness of a solution?
- Why can two equations be added together to get another true equation?
- How can the equations in the form of $x^2 = y$ and $x^3 = y$ where y is a positive rational number be evaluated?
- What is the significance of scientific notation for very large or very small numbers within problem solving situations?

FUNCTIONS (F)
4.F.1 / 4.F.2 / 4.F.3 / 4.F.4 / 4.F.5
Essential Understandings: • A function is a specific topic of relationship in which each input has a unique output that can be represented in a table. • A function can be represented graphically using ordered pairs that consist of the input and the output of the function in the form (input, output). • A function can be represented with an algebraic rule. • The equation $y = mx + b$ is a straight line and that slope and y-intercept are critical to solving real problems involving linear relationships. • Changes in varying quantities are often related by patterns that can be used to predict outcomes and solve problems. • Linear functions may be used to represent and generalize real situations.
Essential Questions: • How do ordered pairs on coordinate graphs help define relationships? • What defines a function and how can it be represented? • What makes a function linear? • How can linear relationships be modeled and used in real-life situations? • Why is one variable dependent upon the other(s) in relationships?
GEOMETRY (G)
4.G.1 / 4.G.2 / 4.G.3 / 4.G.4 / 4.G.5 / 4.G.6 / 4.G.7 / 4.G.8 / 4.G.9 / 4.G.10 / 4.G.11 / 4.G.12 / 4.G.13 / 4.G.14 / 4.G.15 / 4.G.16 / 4.G.17 / 4.G.18 / 4.G.19
Essential Understandings: • Scale drawings can be applied to problem solving situations involving geometric figures. • Geometrical figures can be used to reproduce a drawing at a different scale • The coordinate plane is a tool for modeling real-world and mathematical situations and for solving problems. • Graphing objects in a four quadrant graph can provide ways to measure distances and identify that shapes have specific properties. • Volume of a rectangular prism can be determined by multiplying the length, width and height dimensions when the dimensions are fractional lengths. • Algebraic reasoning is applied when solving geometric problems (i.e., circumference and area of a circle). • Unit rates can be explained in graphical representation, algebraic equations, and in geometry through similar triangles. • Area, volume and surface area are measurements that relate to each other and apply to objects and events in our real life experiences.

Essential Understandings (continued):

- Properties of two-dimensional shapes are used in solving problems involving three-dimensional shapes.
- Two- and three-dimensional shapes and spaces are defined by their properties; real world and geometric structures are composed of these shapes and spaces.
- Planes that cut polyhedra create related two-dimensional figures. Reflections, translations, and rotations are actions that produce congruent geometric objects.
- Dilations, translations, rotations and reflections can be shown using two-dimensional figures on a coordinate plane.
- A dilation is a transformation that changes the size of a figure but not the shape.
- Two similar figures are related by a scale factor, which is the ratio of the lengths of corresponding sides.
- If the scale factor of a dilation is greater than 1, the image resulting from the dilation is an enlargement, and if the scale factor is less than 1, the image is a reduction; both transformations result in similar figures.
- A two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of transformations.
- Two shapes are similar if the length of all the corresponding sides are proportional and all the corresponding angles are congruent.
- Congruent figures have the same size and shape (a rigid, fixed relationship). If the scale factor of a dilation is equal to 1, the image resulting from the transformation is a congruent figure.
- When parallel lines are cut by a transversal, corresponding angles, alternate interior angles, alternate exterior angles, and vertical angles are congruent.
- Right triangles have a special relationship among the side lengths that can be represented by a model and a formula.
- The Pythagorean Theorem can be used to find the missing side lengths in a coordinate plane and real-world situations.
- The Pythagorean Theorem and its converse can be proven.
- Rounded object volume can be calculated with specific formulas.
- Pi is necessary when calculating volume of rounded objects.

Essential Questions:

- Why is it important to use precise language and mathematical tools in the study of two-dimensional and three-dimensional figures?
- How can describing, classifying and comparing attributes of two-dimensional shapes (nets) be useful in solving problems in our three-dimensional (dot paper drawings) world?
- How do graphs illustrate proportional relationships?
- How are geometric figures used to reproduce a drawing at a different scale?
- Problems of area of polygons can be solved by composing and decomposing the polygons.
- What models on the coordinate plane are helpful for understanding and quantifying the volume of rectangular prisms?
- How does what we measure influence how we measure?
- How can space be defined through numbers and measurement?
- How does investigating figures help us build our understanding of mathematics?
- How can proportional relationships of congruent and similar figures be used to solve ratio problems?
- How are scale drawings used to compute actual lengths and area?
- What are transformations and what effect do they have on an object?
- What does the scale factor of a dilation convey?
- How can transformations be used to determine congruency or similarity?
- What angle relationships are formed by a transversal intersecting with two parallel lines?
- Why does the Pythagorean Theorem apply only to right triangles?
- How does the knowledge of how to use right triangles and the Pythagorean Theorem enable the design and construction of such structures as a properly pitched roof, handicap ramps to meet code, structurally stable bridges, and roads?
- How do indirect measurement strategies (using the Pythagorean Theorem) allow for the measurement of items in the real world such as playground structures, flagpoles, and buildings?
- How is the volume and/or surface area of various three-dimensional geometric objects determined?

STATISTICS AND PROBABILITY (SP)
4.SP.1 / 4.SP.2 / 4.SP.3 / 4.SP.4 / 4.SP.5 / 4.SP.6 / 4.SP.7 / 4.SP.8 / 4.SP.9 / 4.SP.10 / 4.SP.11 / 4.SP.12 / 4.SP.13 / 4.SP.14 / 4.SP.15 / 4.SP.16 / 4.SP.17
Essential Understandings: • Statistical questions and the answers account for variability in a data set. • The distribution of a data set is described by its center, spread, and overall shape. • Measures of central tendency for a numerical set of data are summaries of the values using a single number. • Bivariate categorical data display frequencies and relative frequencies can be seen in two-way tables. • Measures of variability describe the variation of the values in the data set using a single number. • Statistics provide information about a population (data set) by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. • Random sampling tends to produce representative samples and support valid inferences. • Two data distributions can be compared using visual and numerical representations based upon measures of center and measures of variability to draw conclusions. • The probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. • The probability of a chance event is approximated by collecting data on the chance process that produces it, observing its long-run relative frequency, and predicting the approximate relative frequency given the probability. • A probability model, which may or may not be uniform, is used to find probabilities of events. • Various tools are used to find probabilities of compound events (including organized lists, tables, tree diagrams, and simulations). • Written descriptions, tables, graphs, and equations are useful in representing and investigating relationships between varying quantities. • Different representations (written descriptions, tables, scatter plots, histograms, box and whisker plots, graphs, and equations) of the relationships between varying quantities may have different strengths and weaknesses. • Slope and y-intercept are keys to solving real problems involving linear relationship models of data. • Some data may be misleading based on representation.

Essential Questions:

- What is the value of using different data representations?
- Using measures of central tendency, how are data sets interpreted and analyzed?
- When is one data display better than another? How can data be displayed strategically?
- When is one statistical measure better than another?
- What makes a good statistical question?
- How can two data distributions be compared?
- How can statistics be used to gain information about a sample population?
- How can a random sample be used to draw inferences of a larger population?
- How are probability and the likelihood of an occurrence related and represented?
- How is probability approximated?
- How is a probability model used?
- How are probabilities of compound events determined?
- What relationships can be seen in bivariate categorical data?
- What conclusions can be drawn from data displayed on a graph?
- What do the slope and y-intercept of a line of best fit signify on a graph? What do outliers signify?
- How can graphs, tables, or equations be used to describe patterns and predict subsequent data or outcomes?

NRS Level 5 Overview

Number and Quantity (N)

The Real Number System (RN)

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers

Quantities (Q)

- Reason quantitatively and use units to solve problems

Algebra (A)

Seeing Structure in Expressions (SSE)

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Expressions (APR)

- Perform arithmetic operations on polynomials

Creating Equations (CED)

- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities (REI)

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

Functions (F)

Interpreting Functions (IF)

- Analyze functions using different representations

Building Functions (BF)

- Build a function that models a relationship between two quantities

Linear, Quadratic, and Exponential Models (LE)

- Construct and compare linear, quadratic, and exponential models and solve problems

NRS Level 5 Overview (continued)

Geometry (G)

Congruence (CO)

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry (SRT)

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity

Circles (C)

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors

Expressing Geometric Properties with Equations (GPE)

- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension (GMD)

- Explain volume formulas and use them to solve problems

Modeling with Geometry (MG)

- Apply geometric concepts in modeling situations

NRS Level 5 Overview (continued)

Statistics and Probability (S)

Interpreting Categorical and Quantitative Data (ID)

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions (IC)

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies of circles

Using Probability to Make Decisions (MD)

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

OVERVIEW EXPLANATION OF MATHEMATICS
NRS LEVELS 5 & 6 – Low & High Adult Secondary Education
NRS Level 5 (Grade Levels 9.0 – 10.9)
NRS Level 6 (Grade Levels 11.0 – 12.9)

Specific information for ASE Level Math
There are two schools of thought related to math instruction in high school. The two schools of thought are: traditional (Algebra I, Geometry, Algebra II – Trigonometry) and integrated (Math I, Math II, Math III). This document, as well as the Illinois ABE/ASE Mathematics Content Standards, follows the traditional path of mathematical instruction and practice.

Difference between Adult Education and High School Education
The literacy through low intermediate standards presented in this document provide for a fairly sequential progression of mathematics instruction and learning. Low and high ASE level standards together prepare adult education students for the rigors of career and/or college level mathematics.

The low and high ASE level standards are grouped into six conceptual categories, each of which is further divided into domain groupings. Adult educators should be aware that specific order, as with all standards, should be left up to individual teachers and programs, not necessarily having to complete all of Level 5 before moving on to Level 6.

College and Career Ready
Low and high ASE standards specify the mathematics that all students should study in order to be college and career ready[8].

It should also be noted that the goal is to be college ***and*** career ready, and levels 5 and 6 represent the entirety of this goal. Level 5 is not exclusively for career preparedness, and Level 6 is not exclusively college ready.

Notes on Courses and Transitions
A major transition for adults is the transition from adult education to post-secondary education for college and/or careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics. Important standards for college and career readiness are distributed across levels and courses. Some of the highest priority content for college and career readiness comes from NRS level 4 (5.0 – 8.9). This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative

[8] Massachusetts Curriculum Framework for Mathematics, 2011, www.doe.mass.edu/frameworks/current

fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume.

Explanation of Conceptual Categories

The Illinois ABE/ASE Mathematics Standards are listed in conceptual categories:

I. Number and Quantity
II. Algebra
III. Functions
IV. Modeling
V. Geometry
VI. Statistics and Probability

These six conceptual categories portray a coherent view of Adult Secondary Education based on the Illinois ABE/ASE Mathematics Content Standards; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

This Curriculum Guide complements the Illinois ABE/ASE Mathematics Content Standards. More information on the above can be found there.

NRS Level 5 – Low Adult Secondary Education
(Grade Levels 9.0 – 10.9)

NUMBER AND QUANTITY (N)
The Real Number System (RN)
5.N.RN.1 / 5.N.RN.2 / 5.N.RN.3
Essential Understandings: • Rational expressions can be written equivalently using rational exponents. • Properties of integer exponents may be applied to expressions with rational exponents. • Adding and multiplying two rational numbers results in a rational number. • The result of adding a rational number and an irrational number is an irrational number. • The result of multiplying a non-zero rational number to an irrational number is an irrational number.
Essential Questions: • How can radical and rational exponents be written equivalently? • How do the properties of integer exponents apply to rational exponents? • What type of number results when adding or multiplying two rational numbers? • What type of number results when adding a rational number to an irrational number? • What type of number results when multiplying a non-zero rational number to an irrational number?
Quantities (Q)
5.N.Q.1
Essential Understandings: • Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations. • Arithmetic and algebra can be used together, with the rules of conversion to transform units. • Scales, graphs, and other data models can be interpreted.
Essential Questions: • When is it advantageous to represent relationships between quantities symbolically? numerically? graphically? • How can the units used in a problem help determine a solution strategy? • How can units, scale, data displays and levels of accuracy be chosen to appropriately represent a situation?

ALGEBRA (A)
Seeing Structure in Expressions (SSE)
5.A.SSE.1 / 5.A.SSE.2 / 5.A.SSE.3
Essential Understandings: • Identify and interpret the different parts of expressions that represent certain values contextually. • Exponential expressions represent a quantity in terms of its context. • Exponential expressions have equivalent forms that can reveal new information to aid in solving problems. • The factors of a quadratic expression/equation can be used to reveal the zeros of the quadratic. • There are several ways to solve a quadratic expression (square roots, completing the square, quadratic formula, and factoring), and that the most efficient route to solving can often be determined by the initial form of the equation. • Quadratic expressions have equivalent forms that can reveal new information to aid in solving problems.
Essential Questions: • What new information will be revealed if this expression is written in a different but equivalent form? • What are the different ways to represent an exponential expression? • What do the factors of a quadratic reveal about the expression? • How can an appropriate expression be created to model data or situations given within context?
Arithmetic with Polynomials and Rational (APR)
5.A.APR.1
Essential Understanding: • Polynomial expressions can be added, subtracted, and multiplied to produce new polynomials.
Essential Question: • How do the arithmetic operations on numbers extend to polynomials?

Creating Equations (CED)
5.A.CED.1 / 5.A.CED.2 / 5.A.CED.3 / 5.A.CED.4
Essential Understandings: • Linear models can be created, used, and interpreted for real-life situations. • Real world situations can be modeled by systems of linear equations. • A system of equations can have no, one, or infinitely many solutions. • Solutions of systems of inequalities are ordered pairs that satisfy all equations as well as inequalities that are often represented by a region. • Exact or approximate solutions can be found using tables, graphs, and/or algebraic manipulations. • Multiple methods may be used to solve a system of equation or inequalities. • Functions can be created to best fit data represented on various models. • Polynomial functions have key features that can be represented on a graph and can be interpreted to provide information to describe relationships of two quantities. These functions can be compared to each other or other functions to model a situation. • Systems can be solved graphically, algebraically or from a table.
Essential Questions: • What real world situations can be modeled by a linear relationship? • How can technology help to determine whether a linear model is appropriate in a given situation? • How can systems of linear equations or inequalities be used to model real world situations? • How can the solution(s) of a system be represented and interpreted? • What processes may be used to solve a system of equations or inequalities? • How can a linear function be found that best fits data from various models? • What are the different methods that can be used to find the solutions of a system of equations? • When changes are made to an equation, what changes are made to the graph? • What new information will be revealed if a formula is written in a different but equivalent form? • How can the solution(s) of a system be represented and interpreted?

Reasoning with Equations and Inequalities (REI)
5.A.REI.1 / 5.A.REI.2 / 5.A.REI.3 / 5.A.REI.4 / 5.A.REI.5 / 5.A.REI. 6

Essential Understandings:

- Algebraic concepts can be proven, and actions taken to arrive at a solution can be justified.
- The relationships between quantities can be explained or justified verbally in the exploration of real world situations.
- The graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which are points that either lie along a line (discrete) or form a line (continuous).
- Linear functions can be represented by a table, graph, verbal description or equation and that each representation can be transferred to another representation.
- Applied problems using quadratics can be answered by either solving a quadratic equation or re-writing the quadratic in a more useful form (factoring to find the zeros, or completing the square to find the maximum or minimum, for instance).
- There are several ways to solve a quadratic equation (square roots, completing the square, quadratic formula, and factoring), and that the most efficient route to solving can often be determined by the initial form of the equation.
- The quadratic formula is derived from the process of completing the square.
- Complex numbers exist and can arise in the solutions of quadratic equations.
- A quadratic function that does not intersect the x-axis has complex zeros.
- The relationship between the factors of a quadratic and the x-intercepts of the graph of the quadratic.

Essential Questions:

- Why are procedures and properties necessary when manipulating numeric or algebraic expressions?
- How can the structure of an equation or an inequality help determine a solution strategy?
- What are complex numbers, and why do they exist?
- How can a quadratic equation be solved?
- How do the factors of a quadratic determine the x-intercepts of the graph and vice versa?
- How is the quadratic formula derived?

FUNCTIONS (F)
Interpreting Functions (IF)
5.F.IF.1 / 5.F.IF.2
Essential Understandings: • The graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which are points that either lie along a line (discrete) or form a line (continuous). • The zeros of each factor of a polynomial determine the x-intercepts of its graph. • Applied problems using quadratics can be answered by either solving a quadratic equation or re-writing the quadratic in a more useful form (factoring to find the zeros, or completing the square to find the maximum or minimum, for instance).
Essential Questions: • How can a function and its notation be used, interpreted and defined? • How can you represent a function and what are the key features of each representation? • What are the key features of a linear or quadratic function? Slope? Intercepts? Maxima? Minima? • What type of linear, quadratic or exponential function is best to model a given situation?
Building Functions (BF)
5.F.BF.1 / 5.F.BF.2
Essential Understandings: • Arithmetic sequences follow a discrete linear pattern, and the common difference is the slope of the line. • Arithmetic sequences are functions with a domain that is a subset of the integers and can be identified by the constant difference between consecutive terms.
Essential Questions: • What is an arithmetic sequence and how does it relate to linear functions? • What is the relationship between recursive and explicit equations and how are they represented symbolically? • How can applied problems using quadratics be answered by either solving a quadratic equation or re-writing the quadratic in a more useful form (e.g., factoring to find the zeros, or completing the square to find the maximum or minimum)?

Linear, Quadratic, and Exponential Models (LE)
5.F.LE.1
Essential Understandings: • Discrete and continuous functions have properties that appear differently when graphed. • Arithmetic and geometric sequences that have a domain of integers, but arithmetic sequences have equal intervals (common difference) and geometric sequences have equal factors (constant ratio). • Arithmetic and geometric sequences can be represented by both recursive and explicit formulas.
Essential Questions: • What type of linear, quadratic or exponential function is best to model a given situation? • How can you decide what type of sequence or function is represented? • What are the different ways you can represent an exponential function? • How do you create an appropriate function to model data or situations given within context? • What new information will be revealed if this equation is written in a different but equivalent form?

GEOMETRY (G)
Congruence (CO)
5.G.CO.1 / 5.G.CO.2 / 5.G.CO.3 / 5.G.CO.4 / 5.G.CO.5 / 5.G.CO.6 / 5.G.CO.7 / 5.G.CO.8 / 5.G.CO.9 / 5.G.CO.10 / 5.G.CO.11 / 5.G.CO.12 / 5.G.CO.13

Essential Understandings:

- The geometric relationships that come from proving triangles congruent or from proving triangles similar may be used to prove relationships between geometric objects represented in the coordinate plane.
- Any two geometric figures are congruent if there is a sequence of rigid motions (rotations, reflections, or translations) that carries one onto the other.
- A proof consists of a hypothesis and conclusion connected with a series of logical steps.
- The basic building blocks of geometric objects are formed from the undefined notions of point, line, distance along a line, and distance around a circular arc.
- Two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles of the triangles are congruent.
- It is possible to prove two triangles congruent without proving corresponding pairs of sides and corresponding pairs of angles of the triangle are congruent if certain subsets of these six congruence relationships are known to be true (e.g. SSS, SAS, ASA, but not SSA).
- Different observed relationships between lines, between angles, between triangles, and between parallelograms are provable using basic geometric building blocks and previously proven relationships between these building blocks and between other geometric objects.
- The geometric relationships that come from proving triangles congruent may be used to prove relationships between geometric objects.
- Geometric figures can be constructed using various tools, methods and relationships.

Essential Questions:

- In terms of rigid motions, when are two geometric figures congruent?
- What are the undefined building blocks of geometry and how are they used?
- What are possible conditions that are necessary to prove two triangles congruent?
- What are the roles of hypothesis and conclusion in a proof?
- What criteria are necessary in proving a theorem?
- What is the significance of demonstrating the relationships between geometric figures through constructions?

Similarity, Right Triangles, and Trigonometry (SRT)
5.G.SRT.1 / 5.G.SRT.2 / 5.G.SRT.3 / 5.G.SRT.4 / 5.G.SRT.5
Essential Understandings: • The geometric relationships that come from proving triangles congruent or from proving triangles similar may be used to prove relationships between geometric objects represented in the coordinate plane. • A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged, and that the dilation of a line segment is longer or shorter in the ratio given by the scale factor of the dilation. • Two geometric figures are similar if there is a sequence of similarity transformations (dilation along with rotations, reflections, or translations) that carries one onto the other. • Two triangles are similar if and only if corresponding pairs of angles are congruent and corresponding pairs of sides are proportional. • It is possible to prove two triangles similar by proving that two pairs of corresponding angles of the triangles are congruent. • Different observed relationships between geometric objects are provable using basic geometric building blocks and previously proven relationships between • these building blocks and between other geometric objects. • The geometric relationships that come from proving triangles congruent or from proving triangles similar may be used to prove relationships between • geometric objects. • The properties of congruent and of similar triangles can be used to solve problems that either involve or can be modeled with triangles.
Essential Questions: • What are the properties of dilations? • In terms of similarity transformations, when are two geometric figures similar? • What are the necessary conditions to know when two triangles are similar? • What are the sufficient conditions to know that two triangles are similar? • How can the Pythagorean Theorem be proven using the geometric relationships that come from proving triangles similar? • How can the geometric relationships that come from proving triangles congruent or from proving triangles similar be applied in problems solving situations?

Circles (C)
5.G.C.1 / 5.G.C.2 / 5.G.C.3 / 5.G.C.4 .5.G.C.5
Essential Understandings: • Different relationships among inscribed angles, radii, and chords of a circle, and between the angles of a quadrilateral inscribed in a circle are provable using previously proven relationships between geometric objects.
Essential Questions: • What are the different relationships among inscribed angles, radii, and chords of a circle, and of the angles of a quadrilateral inscribed in a circle? • What is the relationship between the length of the arc of a circle, the central angle of the circle that intercepts the arc, and the radius of the circle? • What is the area of a sector of a circle?
Expressing Geometric Properties with Equations (GPE)
5.G.GPE.1 / 5.G.GPE.2 / 5.G.GPE.3
Essential Understandings: • Geometric figures can be represented in the coordinate plane. • That algebraic properties (including those related to the distance between points in the coordinate plane) may be used to prove geometric relationships. • The distance formula may be used to determine measurements related to geometric objects represented in the coordinate plane (e.g., the perimeter or area of a polygon. • The algebraic relationship between the slopes of parallel lines and the slopes of perpendicular lines.
Essential Questions: • What is the relationship between the slopes of parallel lines and of perpendicular lines? • Given a polygon represented in the coordinate plane, what is its perimeter and area? • How can geometric relationships be proven through the application of algebraic properties to geometric figures represented in the coordinate plane?

Geometric Measurement and Dimension (GMD)
5.G.GMD.1 / 5.G.GMD.2
Essential Understanding: • The formulas for circumference, area, surface area, and volume of two- and three-dimensional geometric figures can be seen as linear and other functions of the radius.
Essential Question: • How can familiar formulas for two-and three-dimensional geometric figures be viewed as a function and/or model?
Modeling with Geometry (MG)
5.G.MG.1 / 5.G.MG.2
Essential Understandings: • Different geometric objects can be used to model the same or various physical phenomena and the object chosen to model the phenomena will be dependent upon how the model is to be used. • The concept of density and how it may be applied in modeling problems involving area or volume.
Essential Questions: • How can geometric properties and relationships be applied to solve problems that are modeled by geometric objects? • What is density as it relates to area or volume?

STATISTICS AND PROBABILITY (S)
Interpreting Categorical and Quantitative Data (ID)
5.S.ID.1 / 5.S.ID.2 / 5.S.ID.3 / 5.S.ID.4 / 5.S.ID.5 / 5.S.ID.6 / 5.S.ID.7 / 5.S.ID.8 / 5.S.ID.9

Essential Understandings:

- Data can be represented and interpreted in a variety of formats (dot plots, histograms, and box plots).
- Extreme data points (outliers) can skew interpretations of a set of data.
- Synthesizing information from multiple sets of data results in evidence-based interpretation.
- Center and spread of a data set may be compared in multiple ways.
- Data in a two-way frequency table can be summarized using relative frequencies in the context of the data.
- A line of best fit can be generated for a set of data to model the relationship between two variables by hand or with technology.
- A line of best fit aims to minimize the vertical distances between the data points and the points on the line and may be used to make predictions within the proximity of the data.
- Making predictions for values within or near the data set is more reliable than for values far beyond the data set.
- Correlation does not imply causation.
- Exponential functions, like linear, can be used to model real-life situations.
- Key features in graphs and tables shed light on relationships between two quantities.
- Differences between linear and exponential functions, thus allowing them to use the appropriate model.
- Units, scale, data displays, and levels of accuracy represented in situations.
- Functions can be created to best fit data represented on a scatter plot.
- Computations and interpretations are used to decide if differences between parameters are significant.
- A scatter plot may be used to represent data with two quantitative variables and determine how the variables are related.
- The mean and standard deviation of a data set is used to fit a normal distribution.
- Statistics is a process of making inferences.
- Different data collection methods are appropriate for different situations and randomization relates to each.
- Functions have key features that can be represented on a graph and can be interpreted to provide information to describe relationships of two quantities. These functions can be compared to each other or other functions to model a situation.
- Exponential functions can be determined from data and used to represent many real-life situations (population growth, compound interest, depreciation, etc.).
- The properties of a situation or data set determine what type of function (e.g., linear, quadratic, exponential, polynomial, rational, or logarithmic) should be used to model it.

Essential Questions:

- What is the role of statistics in real-world situations?
- When is it appropriate to question the results from a model compared to real-life situations?
- Which data collection method is best used for a specific context?
- How does randomization relate to a data collection method?
- How is a population mean estimated from data from a sample survey?
- When is the difference between parameters significant?
- From a scatterplot, how are two quantitative variables related?
- How is a data set fit to a normal curve?
- How do various representations of data lead to different interpretations of the data?
- When and how can extreme data points impact interpretation of data?
- Why are multiple sets of data used?
- How are center and spread of data sets described and compared?
- How is a data set represented in a two-way frequency table summarized?
- When is it appropriate to use causation or correlation?
- How can computations and interpretations help to determine which model is appropriate in a given situation?
- What are the key features of a linear, quadratic, or exponential function in a modeling situation?
- How can a situation best be modeled by a linear, quadratic, or exponential function?
- How are units, scale, data displays, and levels of accuracy selected to appropriately represent a situation?
- How can a function that best fits the data from a scatter plot be determined?
- How can a scatter plot that is created or interpreted from data fit a function?
- What are key characteristics to identify when choosing a function to model a given situation?

Making Inferences and Justifying Conclusions (IC)
5.S.IC.1 / 5.S.IC.2 / 5.S.IC.3 / 5.S.IC.4 / 5.S.IC.5 / 5.S.IC.6
Essential Understandings: • Statistics can be a tool for making inferences about population versus sample parameters. • Results from a model may or may not be consistent with real-life situations of the process. • Different data collection methods are appropriate for different situations and randomization relates to each. • Data from a sample survey are used to estimate a population mean. • Real-life situations are used to decide if differences between parameters are significant. • A scatter plot may be used to represent data with two quantitative variables and determine how the variables are related. • The mean and standard deviation of a data set are used to fit a normal distribution. • Every day decisions are made based on data collection and interpretation.
Essential Questions: • How can statistics be used to understand parameters of a population versus the sample population? • When is it appropriate to question the results from a model compared to real-life situations? • Which data collection method is best used for a specific context? • How does randomization relate to a data collection method? • How is a population mean estimated from data from a sample survey? • From a scatterplot, how are two quantitative variables related? • How is a data set fit to a normal curve? • How can reports or publications be evaluated based on the data presented?
Using Probability to Make Decisions (MD)
5.S.MD.1 / 5.S.MD.2 / 5.S.MD.3 / 5.S.MD.4 / 5.S.MD.5 / 5.S.MD.6 / 5.S.MD.7
Essential Understanding: • Written descriptions, tables, graphs, and equations are useful in representing and investigating decision-making relationships in everyday life and work.
Essential Question: • How are written descriptions, tables, graphs, and equations used in representing and investigating decision-making relationships in everyday life and work?

NRS Level 6 Overview

Please see *Overview Explanation of Mathematics NRS Levels 5 & 6* for further information.

Number and Quantity (N)

Quantities (Q)

- Reason quantitatively and use units to solve problems

The Complex Number System (CN)

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities (VM)

- Represent and model with vector quantities
- Perform operations on vectors
- Perform operations on matrices and use matrices in applications

Algebra (A)

Seeing Structure in Expressions (SSE)

- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Expressions (APR)

- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Reasoning with Equations and Inequalities (REI)

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve systems of equations
- Represent and solve equations and inequalities graphically

NRS Level 6 Overview (continued)

Functions (F)

Interpreting Functions (IF)

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Building Functions (BF)

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models (LE)

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Trigonometric Functions (TF)

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Geometry (G)

Similarity, Right Triangles, and Trigonometry (SRT)

- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Expressing Geometric Properties with Equations (GPE)

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension (GMD)

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry (MG)

- Apply geometric concepts in modeling situations

NRS Level 6 Overview (continued)

Statistics and Probability (S)

Conditional Probability and the Rules of Probability (CP)

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NRS Level 6 – High Adult Secondary Education
(Grade Levels 11.0 – 12.9)

NUMBER AND QUANTITY (N)
Quantities (Q)
6.N.Q.1 / 6.N.Q.2
Essential Understandings: • Relationships can be represented quantitatively using appropriate units to solve problems in the exploration of real-world situations. • Quantitative models can be created, used, and interpreted to solve real-life situations by using appropriate units.
Essential Questions: • When is it advantageous to represent relationships between quantities numerically? • Why are procedures and properties necessary when manipulating numeric expressions? • What real world situations can be modeled by using a numerical quantity and an appropriate unit? • What are complex numbers, and when might they appear in mathematical problems?
The Complex Number System (CN)
6.N.CN.1 / 6.N.CN.2 / 6.N.CN.3 / 6.N.CN.4 / 6.N.CN.5 / 6.N.CN.6 / 6.N.CN.7 / 6.N.CN.8 / 6.N.CN.9
Essential Understandings: • Arithmetic operations can be performed with complex numbers in standard form ($a + bi$) • Complex numbers exist and can arise in mathematical representations of real-world situations. • Every complex number has a conjugate and it can be used to solve expressions. • Complex numbers are subject to the commutative, associative, and distributive properties. • The relationship between the real and complex factors of a quadratic equation. • There is at least one complex zero in every polynomial function of a positive degree with complex coefficients.

Essential Questions:

- What are complex numbers, and when might they appear in mathematical expressions?
- Which arithmetic operation can be used to create an appropriate complex number to model a given situation?
- How can complex numbers be represented in the rectangular and polar coordinate systems?
- What changes are made to a complex number to find its conjugate?
- Using the relationship $i^2 = -1$, how can the commutative, associative, and distributive properties be used in the arithmetic operations of complex numbers?
- How can complex numbers be used to solve a quadratic equation with real coefficients?
- What is the relationship between the real and complex factors of a quadratic equation and the x-intercepts of a graph of the quadratic?

Vectors and Matrix Quantities (VM)

6.N.VM.1 / 6.N.VM.2 / 6.N.VM.3 / 6.N.VM.4 / 6.N.VM.5 / 6.N.VM.6 / 6.N.VM.7 / 6.N.VM.8 / 6.N.VM.9 / 6.N.VM.10 / 6.N.VM.11 / 6.N.VM.12

Essential Understandings:

- Directed line segments and appropriate symbols are used to represent and solve velocity and other quantities that represent a vector.
- Vector components are found by subtracting the initial point from the coordinates of a terminal point
- The operations of addition, subtraction and multiplication can be applied to vectors.
- Matrices can be used to represent and manipulate data.
- The operations of addition, subtraction and multiplication can be applied to matrices of appropriate dimensions.
- Knowledge of the zero and identity matrices, as well as the determinant, can be applied in matrix addition and multiplication.
- New matrices can be produced by multiplying matrices by scalars.
- Matrices can be used in the transformation(s) of a vector.

Essential Questions:

- What is the purpose of recognizing and writing vectors quantities, having both a magnitude and direction?
- How can vectors represent vector quantities as directed line segments?
- How can the components of a vector be found?
- How can vectors involving velocity and other quantities be represented?
- What would be the result be if vectors were added, subtracted and/or multiplied?
- How are scalars used in matrix multiplication?
- Based upon what is known about matrices, how would the addition, subtraction and multiplication of two matrices be performed and explained?
- What is the role of the zero and identity matrices in matrix addition and multiplication?
- When is the determinant of a square matrix nonzero?
- How can matrices be used in the transformation of vectors?

ALGEBRA (A)

Seeing Structure in Expressions (SSE)

6.A.SSE.1 / 6.A.SSE.2

Essential Understandings:

- The different parts of expressions can represent certain values in the context of a situation and help determine a solution process.
- Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform expressions.
- Equivalent forms of an expression can be found, dependent on how the expression is used.
- Geometric sequences have a domain of integers with equal factors (constant ratios).
- Arithmetic sequences have equal intervals (common difference).
- Geometric sequences can be represented by both recursive and explicit formulas.
- Expressions represent a quantity in terms of its context.
- Expressions have equivalent forms that can reveal new information to aid in solving problems.
- Exponential expressions, like linear expressions, can be used to model real-life situations.
- Differences between linear and exponential expressions allow students to use the appropriate model.

Essential Questions: • How are expressions used to solve real world problems? • When is it advantageous to represent relationships between quantities symbolically? Numerically? • Why are procedures and properties necessary when manipulating numeric or algebraic expressions? • How can the structure of expressions help determine a solution strategy? • What new information will be revealed if an expression is written in a different but equivalent form? • What do the key features of an exponential or linear expression represent in a modeling situation? • How is it determined if a situation is best modeled by an exponential or linear expression? • What does completing the square reveal about a quadratic expression?
Arithmetic with Polynomials and Rational (APR)
6.A.APR.1 / 6.A.APR.2 / 6.A.APR.3 / 6.A.APR.4 / 6.A.APR.5 / 6.A.APR.6
Essential Understandings: • Applied problems using quadratic expressions can be answered by either solving or re-writing the quadratic expression in a more useful form (factoring to find the zeroes, or completing the square to find the maximum or minimum, for instance). • There are several ways to solve a quadratic expression (square roots, completing the square, quadratic formula, and factoring), and that the most efficient route to solving can often be determined by the initial form of the expression. • The quadratic formula is derived from the process of completing the square. • Quadratic expressions have equivalent forms that can reveal new information to aid in solving problems. • The Remainder Theorem can be used to determine roots of polynomials • Polynomial and rational expressions can be added, subtracted, and multiplied to produce new polynomials. • The factors of a quadratic can be used to reveal the zeroes of the quadratic. • The process of completing the square can be used to reveal the vertex of the graph of a quadratic expression (and consequently the minimum or maximum of the function). • The degree of a polynomial helps to determine the end behavior of its graph. • The zeroes of each other of a polynomial expression determine the x-intercepts of its graph. • Graphs of rational expressions are often discontinuous, due to values that are not in the domain of the expression. • The long division algorithm for polynomials can be used to determine horizontal or oblique asymptotes of rational expressions.

Essential Questions:

- How can a quadratic expression be simplified?
- How do the factors of a quadratic determine the x-intercepts of the graph and vice versa?
- When a polynomial $p(x)$ is divided by $x - a$, how can its remainder be found?
- How do the arithmetic operations on numbers extend to polynomials?
- What do the factors of a quadratic reveal about the function?
- What does completing the square reveal about a quadratic function?
- What is the graph of a quadratic function? What are its properties?
- How can a rational expression be simplified?

Reasoning with Equations and Inequalities (REI)

6.A.REI.1 / 6.A.REI.2 / 6.A.REI.3 / 6.A.REI.4 / 6.A.REI.5 / 6.A.REI.6

Essential Understandings:

- The different parts of an expression, simple rational and radical equations, inverse matrices (if it exists), and inequalities can represent certain values in the context of a situation and help determine a solution process.
- Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities in one and two variables.
- Equivalent forms of an expression can be found, dependent on how the expression is used.
- Real world situations can be modeled by systems of linear equations having no, one, or infinitely many solutions.
- Real world situations of systems of inequalities are ordered pairs that satisfy all inequalities, often represented by a region.
- Exact or approximate solutions can be found using tables, graphs, and/or algebraic manipulations.
- Discrete and continuous functions of the first and second degree have properties that appear differently when graphed.
- Exponential expressions represent a quantity in terms of its context and have equivalent forms that can reveal new information to aid in solving problems.
- Exponential functions can be determined from data and used to represent many real-life situations (e.g., population growth, compound interest, depreciation, etc.) by a table, graph, verbal description, or through the use of technology. Each representation can be transferred to another representation.
- Logarithms can be used to solve the exponential equations modeling and can be useful to represent numbers that are very large or that vary greatly and are used to describe real-world situations (e.g., Richter scale, Decibels, pH scale, etc.).

Essential Questions:

- How are various equations, system, and inequalities used to solve real world problems?
- When is it advantageous to represent relationships between quantities symbolically? Numerically? Graphically?
- How can the structure of linear, polynomial, rational, absolute value, exponential, logarithmic, expressions, equations, inequalities help determine a solution strategy?
- How can the solution(s) of a system be represented and interpreted?
- What is the relationship between recursive and explicit equations and how are they represented symbolically?
- How can technology help to determine whether a linear, polynomial, rational, absolute value, exponential, or logarithmic model is appropriate in a given situation?

FUNCTIONS (F)

Interpreting Functions (IF)

6.F.IF.1 / 6.F.IF.2 / 6.F.IF.3 / 6.F.IF.4 / 6.F.IF.5 / 6.F.IF.6 / 6.F.IF.7 / 6.F.IF.8 / 6.F.IF.9

Essential Understandings:

- Functions and its notation have exactly one output for each input and can be defined explicitly or recursively.
- Given a particular representation (such as an equation) of a function, other representations (such as graphs or tables) can be generated and explored.
- Functions (square root, cube root, piecewise, polynomial, rational, exponential, and logarithmic) exhibit key features that can be identified and used to compare functions or to determine solutions to real world experiences.
- Average rate of change can be calculated, estimated and/or interpreted from multiple representations of a function.
- Sequences are functions with a domain that is a subset of the integers and can be identified by the constant difference between consecutive terms.
- Graphs of rational functions are often discontinuous, due to values that are not in the domain of the function.
- That $\log_b y = x$ is another way of expressing $b^x = y$ and that this logarithmic expression can be used to determine the solution of an equation where the unknown is in the exponent.
- The graphs of various functions have key features, including domain, intercepts, where the function is increasing or decreasing (positive or negative) behavior, relative maximums and minimums, symmetries, and end behavior.

Essential Questions:

- What are various representations of a function and how can they be interpreted?
- How are key features of a function identified and explained in relation to the context?
- How are functions and their properties including the increasing or decreasing (positive or negative) behavior, relative maximums and minimums, symmetries, and end behavior compared?
- What determines the type of sequence or function is represented in a real-world situation?
- What are the different ways an exponential function be represented?
- What are the key features of a function or graph and how is it best modeled?
- How is the domain of a rational function related to its graph?
- How can rewriting the equation of a rational function (using long division of polynomials) give further information about its graph?

Building Functions (BF)

6.F.BF.1 / 6.F.BF.2 / 6.F.BF.3 / 6.F.BF.4 / 6.F.BF.5

Essential Understandings:

- Functions with a domain that are a subset of the integers and can be identified by the constant difference between consecutive terms (arithmetic sequences).
- Arithmetic sequences follow a discrete linear pattern, and the common difference is the slope of the line.
- Geometric sequences can be represented by both recursive and explicit formulas.
- Units, scales, data displays, and levels of accuracy are represented in real-world situations.
- Find, understand, and solve the inverse and composite relationship of functions.

Essential Questions:

- What is an arithmetic or geometric sequence and how does it relate to a function?
- What is the relationship between recursive and explicit equations and how are they represented symbolically?
- Which type of arithmetic or geometric sequence or function models a situation?
- How do you choose units, scale, data displays and levels of accuracy to appropriately represent a situation?
- How can the inverse and composite relationship of a function be used in a real-world situation?

Linear, Quadratic, and Exponential Models (LE)

6.F.LE.1 / 6.F.LE.2 / 6.F.LE.3 / 6.F.LE.4

Essential Understandings:

- Differences between linear and exponential functions allow these functions to be used as an appropriate model.
- Use graphs and tables to recognize a situation where a constant grows or decays by a constant percent rate.
- Interpret the parameters in a linear or exponential function in terms of a real-world context (e.g., compounding returns or investment goals).

Essential Questions:

- What are the different ways an exponential or linear function can be compared?
- How can the parameters of a linear or exponential function be interpreted?
- How is it determined when a situation is best modeled by an exponential or linear function?

Trigonometric Functions (TF)

6.F.TF.1 / 6.F.TF.2 / 6.F.TF.3 / 6.F.TF.4 / 6.F.TF.5 / 6.F.TF.6 / 6.F.TF.7 / 6.F.TF.8 / 6.F.TF.9

Essential Understandings:

- The unit circle allows all real numbers to work in trigonometric functions.
- Pythagorean identities can be proven and used to solve problems with specified context.
- Key features in a unit circle shed light on the relationships between two quantities.
- Trigonometric functions can be represented by a table, graph, verbal description or equation, and each representation can be transferred to another representation.
- Specific transformations occur to trigonometric functions based on a value *k* and its manipulation to the function.
- The trigonometric functions $\sin(x)$, $\cos(x)$, or $\tan(x)$ can be used to model real-life situations that exhibit periodic behavior.
- Changing parameters such as amplitude, period, and midline of a function will alter its graph and that these parameters are related to the context or phenomena being modeled.
- The trigonometric functions of sine, cosine, and tangent can be used to solve sum or difference problems.
- Using technology, evaluate and interpret inverse functions to solve trigonometric equations arising in real-world situations.
- Use the unit circle to explain symmetry with odd and even and periodicity of trigonometric functions.

Essential Questions:

- How can the unit circle be read and interpreted using radians?
- How does the Pythagorean theorem and the unit circle relate to the identity $\sin^2(x) + \cos^2(x) = 1$?
- What do the key features or characteristics of a trigonometric function represent?
- What are the different ways a trigonometric function can be represented?
- What transformations can occur to a trigonometric function/graph?
- How can the graphs of trigonometric functions be modified to best fit the situations being modeled?
- How do factors such as amplitude, period, midline, and horizontal shift affect these functions and relate to the phenomena being modeled?
- How can the trigonometric functions of sine, cosine, and tangent be used to solve sum or difference problems?
- How can technology be used to evaluate and interpret inverse functions to solve trigonometric equations arising in real-world situations?
- How can the unit circle explain symmetry with odd and even and periodicity of trigonometric functions?

GEOMETRY (G)
Similarity, Right Triangles, and Trigonometry (SRT)
6.G.SRT.1 / 6.G.SRT.2 / G.SRT.3 / 6.G.SRT.4 / 6.G.SRT.5 / 6.G.SRT.6
Essential Understandings: • The ratios of the sides of right triangles are functions of the acute angles of the triangle. • The sine of an acute angle in a right triangle is equal to the cosine of that angle's complement (and vice versa). • The Pythagorean Theorem applies only to right triangles. • Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of the triangle. • Prove the Laws of Sine and Cosine for right triangle trigonometry in real-world situations. • Prove the Laws of Sine and Cosine for non-right triangle trigonometry in real-world situations.
Essential Questions: • How does similarity give rise to the trigonometric ratios? • How do the trigonometric ratios of complementary angles relate to one another? • How can the Pythagorean Theorem be used to solve problems involving triangles? • How can the formula $A = \frac{1}{2}ab\sin(C)$ for the area of the triangle be derived and used? • How can the Laws of Sine and Cosine for right triangle trigonometry in real-world situations be proved? • How can the Laws of Sine and Cosine for non-right triangle trigonometry in real-world situations be proved?
Expressing Geometric Properties with Equations (GPE)
6.G.GPE.1 / 6.G.GPE.2 / 6.G.GPE.3 / 6.G.GPE.4
Essential Understandings: • The equation of a circle can be found given a center and radius length. • The equation of a parabola can be found given a focus and directrix. • The equation of an ellipse and hyperbola can be found given the foci or the sum/difference of distances from the foci. • Use coordinates to prove simple geometric theorems algebraically.
Essential Questions: • How can the equation of a circle be found given a center and radius length? • How can the equation of a parabola be found given a focus and directrix? • How can the equation of an ellipse and hyperbola be found given the foci or the sum/difference of distances from the foci? • Using coordinates, how can simple geometric theorems be proven algebraically?

Geometric Measurement and Dimension (GMD)
6.G.GMD.1 / 6.G.GMD.2 / 6.G.GMD.3
Essential Understandings: • Given an informal argument, explain the formulas for the circumference, area of a circle, volume of a cylinder, pyramid, and cone can be solved. • Given an informal argument using Cavalieri's principle, explain the formulas of a sphere and other solid figures can be solved. • Identify the shapes of two-dimensional cross-sections of three-dimensional objects. • Identify three-dimensional objects generated by rotations of two-dimensional objects.
Essential Questions: • How can an informal argument explain the formulas for the circumference, area of a circle, volume of a cylinder, pyramid, and cone? • How can an informal argument using Cavalieri's principle explain the formulas of a sphere and other solid figures? • How can shapes of two-dimensional cross-sections of three-dimensional objects be identified? • How can shapes of three-dimensional objects generated by rotations of two-dimensional objects be identified?
Modeling with Geometry (MG)
6.G.MG.1
Essential Understanding: • Geometric objects may be used to model various physical phenomena.
Essential Question: • How can geometric figures be used to model physical phenomena or problem situations?

STATISTICS (S)
Conditional Probability and the Rules of Probability (CP)
6.S.CP.1 / 6.C.SP.2 / 6.S.CP.3 / 6.S.CP.4 / 6.S.CP.5 / 6.S.CP.6 / 6.S.CP.7 / 6.S.CP.8 / 6.S.CP.9

Essential Understandings:

- Events can be described as a subset of a sample space.
- The probability of two events occurring together is the product of their probabilities, if and only if then the events are independent.
- The probability of two events can be conditional on each other and the interpretation of that probability.
- Two-way frequency tables can be used to decide if events are independent and to find conditional probabilities.
- Conditional probability and independence are applied to everyday situations.
- Conditional probability of A given B can be found and interpreted in context.
- The Addition or Multiplication Rule can be applied and the resulting probability can be interpreted in a context or in terms of a given model.
- Permutations and combinations can be used to compute probabilities of compound events in problem-solving situations.

Essential Questions:

- How can an event be described as a subset of outcomes using correct set notation?
- How are probabilities, including joint probabilities, of independent events calculated?
- How are probabilities of independent events compared to their joint probability?
- How does conditional probability apply to real-life events?
- How are two-way frequency tables used to model real-life data?
- How are conditional probabilities and independence interpreted in relation to a situation?
- What is the difference between compound and conditional probabilities?
- How is the probability of event (A or B) found?
- How can the Addition or Multiplication Rule be applied and the resulting probability be interpreted within a context or in terms of a given model?
- How can permutations and combinations be used to compute probabilities of compound events in problem-solving situations?