Illinois Community College Board
Adult Education & Literacy

Illinois ABE/ASE Mathematics Model Curriculum

NRS Level 6

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NRS Level 6 Math
High Adult Secondary Education (Grade Levels 11.0 – 12.9)

NUMBER AND QUANTITY (N)
Quantities (Q)
6.N.Q.1 / 6.N.Q.2

Essential Understandings:

- Relationships can be represented quantitatively using appropriate units to solve problems in the exploration of real-world situations.
- Quantitative models can be created, used, and interpreted to solve real-life situations by using appropriate units.

Essential Questions:

- When is it advantageous to represent relationships between quantities numerically?
- Why are procedures and properties necessary when manipulating numeric expressions?
- What real world situations can be modeled by using a numerical quantity and an appropriate unit?
- What are complex numbers, and when might they appear in mathematical problems?

Student will be able to:

- Define, create, use, interpret, and represent quantitative relationships using appropriate units when solving a real-world situation.
- Find a level of accuracy appropriate to the limitations on measurement when reporting quantities.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed Advanced Fire activity and worksheet
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math Journals

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

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This is a lesson outline helping students explore the real-world implications of functions and independent/dependent variables.

How Fast Can a Fire Advance?

1. Let students know what they will be doing and learning.
2. Teacher input is to lead a discussion to prepare students to identify independent and dependent variables in the Advanced Fire exercise.
3. Guide practice by demonstrating how to start activity on the computer. A) Setting and changing the probability that fire will spread. B) Setting a tree on fire and watching it spread.
4. Discuss the fact that Advanced Fire has multiple variables.
5. Independent practice should be done by grouping students into 3 or 4 and asking them to choose variables related to the size of the forest and the proportion of trees left standing.
6. For closure, initiate a class discussion on all findings on their chosen independent and dependent variables. They can also describe the relationships that they found between the two variables.

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Pencil and calculator
- Copy of activity and worksheet from http://shodor.org/interactivate/lessons/AdvancedFire/

List of Technology Resources:

- http://shodor.org/interactivate/lessons/AdvancedFire/
- Teaching Ideas www.teachingideas.co.uk
- Super Teacher www.superteacherworksheets.com
- Kuta Software https://www.kutasoftware.com
- Plato Learning Environment http://ple.platoweb.com/
- Purple Math www.purplemath.com

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The Complex Number System (CN)

6.N.CN.1 / 6.N.CN.2 / 6.N.CN.3 / 6.N.CN.4 / 6.N.CN.5 / 6.N.CN.6 / 6.N.CN.7 / 6.N.CN.8 / 6.N.CN.9

Essential Understandings:

- Arithmetic operations can be performed with complex numbers in standard form ($a + bi$)
- Complex numbers exist and can arise in mathematical representations of real-world situations.
- Every complex number has a conjugate and it can be used to solve expressions.
- Complex numbers are subject to the commutative, associative, and distributive properties.
- The relationship between the real and complex factors of a quadratic equation.
- There is at least one complex zero in every polynomial function of a positive degree with complex coefficients.

Essential Questions:

- What are complex numbers, and when might they appear in mathematical expressions?
- Which arithmetic operation can be used to create an appropriate complex number to model a given situation?
- How can complex numbers be represented in the rectangular and polar coordinate systems?
- What changes are made to a complex number to find its conjugate?
- Using the relationship $i^2=-1$, how can the commutative, associative, and distributive properties be used in the arithmetic operations of complex numbers?
- How can complex numbers be used to solve a quadratic equation with real coefficients?
- What is the relationship between the real and complex factors of a quadratic equation and the *x*-intercepts of a graph of the quadratic?

Student will be able to:

- State what a complex number is, when it might appear in a mathematical expression, and which arithmetic operations can be used to create a complex number.
- Write a complex number in a + b*i* form and represent this number in the rectangular and polar coordinate system.
- Show how the commutative, associative and distributive properties are used in the arithmetic operations of complex numbers by using the relationship $i^2 = -1$.
- Find the conjugate of a complex number written in a + b*i* form and explain how it can be used to solve expressions

- Explain why the rectangular and polar forms of a given complex number represents the same number.
- Represent addition, subtraction, multiplication and the conjugate of complex numbers geometrically on a complex plane, using properties of the representation for computation.
- Find the distance between numbers in the complex plane as the modulus of the difference.
- Find the midpoint of a segment in the complex plane as the average of numbers at its endpoints.
- Use complex numbers to solve quadratic equations with real coefficients.
- Describe the relationships between the real and complex factors of a quadratic equation in terms of the x-intercepts of a graph or the zeros of the function.
- Use the Fundamental Theorem of Algebra to show that it is true for quadratic polynomials.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed worksheet and activity from "Imagining a New Number Learning Task"
- Plato Learning Environment tests-monitor management system by objective and NRS level.

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:
"Imagining a New Number Learning Task" found at lhsblogs.typepad.com

Activating Strategies: (Learners Mentally Active)
Share the historical story of "I" from "Imagining a New Number Learning Task"
Acceleration/Previewing: (Key Vocabulary)
Imaginary form, complex number, "I", standard form, pure imaginary number, complex conjugates, and complex number plane, absolute value of a complex number
Teaching Strategies: (Collaborative Pairs; Distributed Guided Practice; Distributed Summarizing; Graphic Organizers)

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Walk through the "Imagining a New Number Learning Task." Insert the graphic organizers on pages as needed. Use small groups and collaborative pairs to complete the task, including guided practice on teacher-made practice sheets. At the end of each lesson, groups share with the class.
Distributed Guided Practice/Summarizing Prompts: (Prompts Designed to Initiate Periodic Practice or Summarizing)
What exactly is the absolute value of any number? How does the definition of absolute value apply to the complex plane? Does the definition of absolute value also work with real numbers? 22ba+ How do you write a real number as a complex number?
Extending/Refining Strategies:
Solve: $3x^2 - \sqrt{2}x + 2 = 0$ and $\sqrt{2}x^2 - 6x + \sqrt{8} = 0$

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Graph paper
- Handouts from "Imagining a New Number" learning activity - lhsblogs.typepad.com

List of Technology Resources:

- Teaching Ideas www.teachingideas.co.uk
- Plato Learning Environment http://ple.platoweb.com/
- Purple Math www.purplemath.com
- Cool Math www.coolmath.com

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Vectors and Matrix Quantities (VM)
6.N.VM.1 / 6.N.VM.2 / 6.N.VM.3 / 6.N.VM.4 / 6.N.VM.5 / 6.N.VM.6 / 6.N.VM.7 / 6.N.VM.8 / 6.N.VM.9 / 6.N.VM.10 / 6.N.VM.11 / 6.N.VM.12

Essential Understandings:

- Directed line segments and appropriate symbols are used to represent and solve velocity and other quantities that represent a vector.
- Vector components are found by subtracting the initial point from the coordinates of a terminal point
- The operations of addition, subtraction and multiplication can be applied to vectors.
- Matrices can be used to represent and manipulate data.
- The operations of addition, subtraction and multiplication can be applied to matrices of appropriate dimensions.
- Knowledge of the zero and identity matrices, as well as the determinant, can be applied in matrix addition and multiplication.
- New matrices can be produced by multiplying matrices by scalars.
- Matrices can be used in the transformation(s) of a vector.

Essential Questions:

- What is the purpose of recognizing and writing vectors quantities, having both a magnitude and direction?
- How can vectors represent vector quantities as directed line segments?
- How can the components of a vector be found?
- How can vectors involving velocity and other quantities be represented?
- What would be the result be if vectors were added, subtracted and/or multiplied?
- How are scalars used in matrix multiplication?
- Based upon what is known about matrices, how would the addition, subtraction and multiplication of two matrices be performed and explained?
- What is the role of the zero and identity matrices in matrix addition and multiplication?
- When is the determinant of a square matrix nonzero?
- How can matrices be used in the transformation of vectors?

Student will be able to:

- Write vector quantities having both a magnitude and a direction.
- Solve velocity and quantities representing vectors with directed line segments and appropriate symbols.
- Find vector components by subtracting the initial point from the coordinates of a terminal point.
- Use either the end-to-end, component-wise, or the parallelogram rule to add and subtract vectors.

- Find the magnitude and direction form to determine the magnitude and direction of the sum of two vectors.
- Represent vector subtraction graphically by connecting tips in the appropriate order, performing vector subtraction component-wise.
- Define vector subtraction of $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, having the same magnitude as w, but pointing in the opposite direction.
- Multiply a vector by a scalar.
- Represent scalar multiplication graphically, scaling vectors and possibly reversing the direction.
- Find the magnitude of a scalar multiple of cv using $||cv|| = |c|v$, knowing the direction of cv can be either along v (for $c > 0$) or against v (for $c < 0$).
- Use matrices to represent and manipulate data.
- Use scalars in matrix multiplication.
- Add, subtract and multiply matrices.
- Use scalars in matrix multiplication.
- Uses the zero and identity matrix in matrix addition and multiplication.
- Use a determinant of a square matrix, which is zero, if and only if the matrix has a multiplicative inverse.
- Multiply a vector by a one column matrix by a matrix of suitable dimensions to produce another vector, using matrices in the transformation of vectors.
- Use a 2 x 2 matrices as transformations of a plane, interpreting the absolute value of the determinant in terms of area.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed Purple Math matrix and vector activities and problems
- Optional: completed “Vector Voyage!” assignment (including Worksheets 1 and 2)
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson demonstrating how to represent vectors geometrically, how to scale vectors, and how to combine vectors by addition. Encourage students to explore how vectors can be used to solve applied problems
- Use material from PurpleMath.com relating to vectors and matrices to introduce the following concepts:
  - Matrix Definitions
  - Adding and Subtracting Matrices
  - Matrix Multiplication
  - Matrix Row Operations
- Explain that vectors and vector operations are used extensively in navigation on water and air. Display on overhead the website Teach Engineering and visit the lesson "Vector Voyage!" Cover the introductory material as a group and discuss how vectors can be represented geometrically with directed line segments and how vectors and scalar multiples of vectors can be used to model navigation routes
- Assign this "Vector Voyage!" lesson as group work or an extension/homework activity as appropriate

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One–to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Purplemath.com
- Teach Engineering.org

List of Technology Resources:

- Purple Math Advanced Algebra Topics www.purplemath.com
- Teach Engineering "Vector Voyage!" Activity https://www.teachengineering.org/activities/view/cub_navigation_lesson02_activity1

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High Adult Secondary Education (Grade Levels 11.0 – 12.9)

ALGEBRA (A)
Seeing Structure in Expressions (SSE)
6.A.SSE.1 / 6.A.SSE.2

Essential Understandings:

- The different parts of expressions can represent certain values in the context of a situation and help determine a solution process.
- Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform expressions.
- Equivalent forms of an expression can be found, dependent on how the expression is used.
- Geometric sequences have a domain of integers with equal factors (constant ratios).
- Arithmetic sequences have equal intervals (common difference).
- Geometric sequences can be represented by both recursive and explicit formulas.
- Expressions represent a quantity in terms of its context.
- Expressions have equivalent forms that can reveal new information to aid in solving problems.
- Exponential expressions, like linear expressions, can be used to model real-life situations.
- Differences between linear and exponential expressions allow students to use the appropriate model.

Essential Questions:

- How are expressions used to solve real world problems?
- When is it advantageous to represent relationships between quantities symbolically? Numerically?
- Why are procedures and properties necessary when manipulating numeric or algebraic expressions?
- How can the structure of expressions help determine a solution strategy?
- What new information will be revealed if an expression is written in a different but equivalent form?
- What do the key features of an exponential or linear expression represent in a modeling situation?
- How is it determined if a situation is best modeled by an exponential or linear expression?
- What does completing the square reveal about a quadratic expression?

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Student will be able to:

- Create an equivalent form of a quadratic expression by completing the square, which will reveal the maximum or minimum of the function it defines.
- Write an expression in a different but equivalent form to obtain new information for an exponential function.
- Use the formula to solve a geometric series to solve real-world situations.
- Derive the formula for the sum of a finite geometric series when the common ration is not 1.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate solutions to five problems from Laws of Exponents game
- Complete and accurate solutions to “Completing the Square” material
- Plato Learning Environment tests-monitor management system by objective and NRS level

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Certain rules apply to exponents, and students will learn and use these rules to simplify algebraic expressions and build upon them in future lessons
- Review material from PurpleMath.com section “Exponents: Basic Rules”
- Direct practice problems to the whole class, then to partners, and then have students complete some problems independently
- Tell students they will practice the laws of exponents with a small group game. Students take turns until everyone in the group has gotten at least five turns. How to play the game:
    - Divide students into groups of 3. Have one member from each group get a spinner, a paper clip, and a penny. (Show students how to use their pencil and paper clip as a spinner)
    - Students take turns spinning the spinner to get the first x and exponent. Then they flip the coin to determine the function of division or multiplication. Then they spin the spinner again to get a second x and exponent
    - Give each member of the group a piece of paper and pencil to record their work and answers – to be submitted to teacher after the game

- As a whole group, review the material on PurpleMath.com section "Completing the Square: Solving Quadratic Equations"
- Have students complete a few of the examples as group work to demonstrate competency

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One–to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Exponents: Basic Rules material from PurpleMath.com
- Laws of Exponent Game instructions
- Spinners
- Paper clips
- Pennies
- "Completing the Square: Solving Quadratic Equations" material from PurpleMath.com

List of Technology Resources:

- Plato Learning Environment http://ple.platoweb.com/
- Purple Math www.purplemath.com

Arithmetic with Polynomials and Rational (APR)
6.A.APR.1 / 6.A.APR.2 / 6.A.APR.3 / 6.A.APR.4 / 6.A.APR.5 / 6.A.APR.6

Essential Understandings:

- Applied problems using quadratic expressions can be answered by either solving or re-writing the quadratic expression in a more useful form (factoring to find the zeroes, or completing the square to find the maximum or minimum, for instance).
- There are several ways to solve a quadratic expression (square roots, completing the square, quadratic formula, and factoring), and that the most

efficient route to solving can often be determined by the initial form of the expression.

- The quadratic formula is derived from the process of completing the square.
- Quadratic expressions have equivalent forms that can reveal new information to aid in solving problems.
- The Remainder Theorem can be used to determine roots of polynomials
- Polynomial and rational expressions can be added, subtracted, and multiplied to produce new polynomials.
- The factors of a quadratic can be used to reveal the zeroes of the quadratic.
- The process of completing the square can be used to reveal the vertex of the graph of a quadratic expression (and consequently the minimum or maximum of the function).
- The degree of a polynomial helps to determine the end behavior of its graph.
- The zeroes of each other of a polynomial expression determine the x-intercepts of its graph.
- Graphs of rational expressions are often discontinuous, due to values that are not in the domain of the expression.
- The long division algorithm for polynomials can be used to determine horizontal or oblique asymptotes of rational expressions.

Essential Questions:

- How can a quadratic expression be simplified?
- How do the factors of a quadratic determine the x -intercepts of the graph and vice versa?
- When a polynomial $p(x)$ is divided by $x-a$, how can its remainder be found?
- How do the arithmetic operations on numbers extend to polynomials?
- What do the factors of a quadratic reveal about the function?
- What does completing the square reveal about a quadratic function?
- What is the graph of a quadratic function? What are its properties?
- How can a rational expression be simplified?

Students will be able to:

- Use the Remainder Theorem to determine roots of a polynomial.
- Factor or complete the square to solve a quadratic expression for the zeros (found when factoring), and the maxima or the minima (found when completing the square).
- Find the factors of a quadratic function and determine how these factors relate to the x-intercepts (the zeros) of the graph or vice versa.
- Prove polynomial identities and use them to describe numerical relationships (e.g., generate Pythagorean triples).

- Use the Binomial Theorem for expansion of $(x+y)^n$ in powers of x and y for a positive integer n, with coefficients determined by Pascal's Triangle.
- Rewrite simple rational expressions in different forms, using inspection, long division, or technology.
- Prove that rational expressions form a system analogous to rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression.
- Add, subtract, multiply, and divide rational expressions.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Independent notes and work on Remainder Theorem
- Group and independent work to PurpleMath.com problems for Rational Expressions and Binomial Theorem
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson for students to successfully learn and apply the Remainder Theorem for a polynomial and a number
- Explain why the Remainder Theorem is useful for evaluating polynomials at a given value of x. Students do not necessarily have to understand it as a proof; they need to understand how to use the Theorem.
- Explain that The Remainder Theorem starts with an unnamed polynomial *p(x)* where "*p(x)*" means some polynomial *p* whose variable is "*x*". The Theorem works by dividing that polynomial by some linear factor *x-a*, where "*a*" is a number.
- Start with a concrete example of long polynomial division: p(x)= $x^3 - 7x - 6$ and divide by the linear factor**, x- 4(a=4).**

$X^2 + 4x + 9$

X - 4√ $x^3 + 0x^2 - 7x - 6$

$-x^3 + 4x^2$

$4x^2 - 7x - 6$

$-4x^2 + 16x$

$9x - 6$

$\underline{-9x + 36}$
30

- This produces a quotient of $q(x) = x^2 + 4x + 9$ on top, with a remainder of $r(x) = 30$.
  Remind students that when there is no number in front of x, the degree on x is understood as "1." In polynomial terms, we are dividing by a linear factor, so the remainder must be a constant value, since the degree on x is "1".
- The Remainder Theorem then points out the connection between division and multiplication. For example, since 12÷ 3 = 4, then 4 x 3 = 12. If you get a remainder, you do the multiplication and then add the remainder back in. For example, 13 ÷ 5 = 2 R 3, then 13= 5 x2 + 3. This process works the same way with polynomials. Since $(x^3 - 7x – 6) ÷ (x- 4) = x^2 + 4x + 9$ with a remainder of 30, then $x^3 – 7x – 6 = (x – 4) (x^2 + 4x + 9) + 30$.
- Present a lesson to students on rational expressions and how to rewrite them using material from the PurpleMath.com Advanced Algebra Topics
- Walk through the lesson titled "Rational Expressions: Simplifying" and ask students to complete some of the sample problems in pairs or independently
- Assign the lesson from PurpleMath.com titled "Binomial Theorem: Formulas" as an extension activity or homework as appropriate

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One–to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:
(core and supplemental)

- PurpleMath.com Rational Expressions and Binomial Theorem lessons

List of Technology Resources:

- Purple Math www.purplemath.com
- Cool Math www.coolmath.com
- Math Planet www.mathplanet.com/

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Reasoning with Equations and Inequalities (REI)

6.A.REI.1 / 6.A.REI.2 / 6.A.REI.3 / 6.A.REI.4 / 6.A.REI.5 / 6.A.REI.6

Essential Understandings:

- The different parts of an expression, simple rational and radical equations, inverse matrices (if it exists), and inequalities can represent certain values in the context of a situation and help determine a solution process.
- Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.
- Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities in one and two variables.
- Equivalent forms of an expression can be found, dependent on how the expression is used.
- Real world situations can be modeled by systems of linear equations having no, one, or infinitely many solutions.
- Real world situations of systems of inequalities are ordered pairs that satisfy all inequalities, often represented by a region.
- Exact or approximate solutions can be found using tables, graphs, and/or algebraic manipulations.
- Discrete and continuous functions of the first and second degree have properties that appear differently when graphed.
- Exponential expressions represent a quantity in terms of its context and have equivalent forms that can reveal new information to aid in solving problems.
- Exponential functions can be determined from data and used to represent many real-life situations (e.g., population growth, compound interest, depreciation, etc.) by a table, graph, verbal description, or through the use of technology. Each representation can be transferred to another representation.
- Logarithms can be used to solve the exponential equations modeling and can be useful to represent numbers that are very large or that vary greatly and are used to describe real-world situations (e.g., Richter scale, Decibels, pH scale, etc.).

Essential Questions:

- How are various equations, system, and inequalities used to solve real world problems?
- When is it advantageous to represent relationships between quantities symbolically? Numerically? Graphically?

- How can the structure of linear, polynomial, rational, absolute value, exponential, logarithmic, expressions, equations, inequalities help determine a solution strategy?
- How can the solution(s) of a system be represented and interpreted?
- What is the relationship between recursive and explicit equations and how are they represented symbolically?
- How can technology help to determine whether a linear, polynomial, rational, absolute value, exponential, or logarithmic model is appropriate in a given situation?

Student will be able to:

- Find the solution of a simple rational and radical equation in one variable, giving examples showing how extraneous solutions may arise.
- Find the solution of a simple linear equation and a quadratic equation in two variables algebraically and graphically.
- Write a system of linear equations as a single matrix equation in a vector variable.
- Write and find the inverse of a matrix, using it to solve systems of linear equations. (Note: Technology may be used.)
- Find the x-coordinates of the points where the graphs of equations $y = f(x)$ and $y = g(x)$ intersect and explain why these are the solutions of the equation $f(x) = g(x)$.
- Using technology, tables of values, or successive approximations to find the exact and approximate solutions for functions which can be linear, polynomial, rational, absolute value, exponential, or logarithmic.
- Graph the solutions of a linear inequality in two variables as a half-plane.
- Graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate notes and work on radical equations
- Complete and accurate solutions to Khan Academy problems on linear systems and linear inequalities
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals

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High Adult Secondary Education (Grade Levels 11.0 – 12.9)

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on Solving Radical Equations
- Review key terms: Radical equation, Radican
- Ask students: What do you know about solving equations? Let's look at an equation with a variable as a radicand: $\sqrt{x} = 6$
- For what value of x would you substitute to make this equation a true statement? Examine the algebra to solve this problem

$\sqrt{x} = 6$ **$\sqrt{x} = \sqrt{36}$** **$x = 36$**	**$\sqrt{x} = 6$** **$(\sqrt{x})^2 = 6^2$** **$x = 36$**	**$\sqrt{x} = 6$** **$X^{1/2} = 6$** **$(x^{1/2})^2 = (6)^2$** **$x = 36$**

- Additional examples to be done as group or independent work:
  1) $\sqrt{x}+5 = 11$ 2) $\sqrt{x} – 4 = 7$ 3) $\sqrt{x}-3 = 0$
- Students can also be taught to solve quantities such as *x-5* under the entire radicand
- Present a lesson on linear equations using material from Khan Academy including: "Solving Linear Systems with Matrix Equations" and "Solving and Graphing Linear Inequalities"
- Students complete some of the sample problems in pairs or independently

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Overhead projector
- Khan Academy material on linear systems and linear inequalities

List of Technology Resources:

- Khan Academy Linear Systems - https://www.khanacademy.org/math/precalculus/precalc-matrices/solving-equations-with-inverse-matrices/v/solving-matrix-equation
- Khan Academy Linear Inequalities - https://www.khanacademy.org/math/algebra-home/alg-basic-eq-ineq/alg-one-step-inequalities/v/solving-and-graphing-linear-inequalities-in-two-variables-1

FUNCTIONS (F)
Interpreting Functions (IF)

6.F.IF.1 / 6.F.IF.2 / 6.F.IF.3 / 6.F.IF.4 / 6.F.IF.5 / 6.F.IF.6 / 6.F.IF.7 / 6.F.IF.8 / 6.F.IF.9

Essential Understandings:

- Functions and its notation have exactly one output for each input and can be defined explicitly or recursively.
- Given a particular representation (such as an equation) of a function, other representations (such as graphs or tables) can be generated and explored.
- Functions (square root, cube root, piecewise, polynomial, rational, exponential, and logarithmic) exhibit key features that can be identified and used to compare functions or to determine solutions to real world experiences.
- Average rate of change can be calculated, estimated and/or interpreted from multiple representations of a function.
- Sequences are functions with a domain that is a subset of the integers and can be identified by the constant difference between consecutive terms.
- Graphs of rational functions are often discontinuous, due to values that are not in the domain of the function.
- That $log_b\, y = x$ is another way of expressing $b^x = y$ and that this logarithmic expression can be used to determine the solution of an equation where the unknown is in the exponent.
- The graphs of various functions have key features, including domain, intercepts, where the function is increasing or decreasing (positive or negative) behavior, relative maximums and minimums, symmetries, and end behavior.

Essential Questions:

- What are various representations of a function and how can they be interpreted?
- How are key features of a function identified and explained in relation to the context?
- How are functions and their properties including the increasing or decreasing (positive or negative) behavior, relative maximums and minimums, symmetries, and end behavior compared?
- What determines the type of sequence or function is represented in a real-world situation?

- What are the different ways an exponential function be represented?
- What are the key features of a function or graph and how is it best modeled?
- How is the domain of a rational function related to its graph?
- How can rewriting the equation of a rational function (using long division of polynomials) give further information about its graph?

Student will be able to:

- Define the domain (from one function set) and range (from another function set), assigning each element of the domain to exactly one element of the range.
- Use f(x) to denote the output of function f corresponding to the input or domain and the graph of function f is the graph of f(x).
- Use appropriate function notation, evaluating functions for inputs in their domains and interpreting statements that use function notation in terms of a context.
- Determine and use the appropriate sequence or function (sometimes recursive) that best models a real world situation.
- Model a relationship between functional quantities, interpreting key features of the function graphs and tables in terms of the quantities.
- Draw a graph expressed symbolically and show key features of the graph, by hand in simple cases and/or using technology for more complicated cases.
- Compare two functions each represented either algebraically, graphically, given tables or by a verbal description in terms of their domains, their intercepts, their increasing and decreasing behavior, their relative maximums and minimums, their symmetries, and their end behaviors.
- Calculate, estimate, or interrupt the average rate of change from multiple representations of a function.
- Represent, interpret, and graph various representations of a function (e.g., the square root function, the cubic function, the piecewise function, the polynomial function, the rational, exponential and logarithmic function) by the process of factoring and completing the square to show zeros, extreme values, and symmetry of the graph, interpreting these concepts in terms of a context.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed and accurate Super Bowl ad activity sheet

  Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals

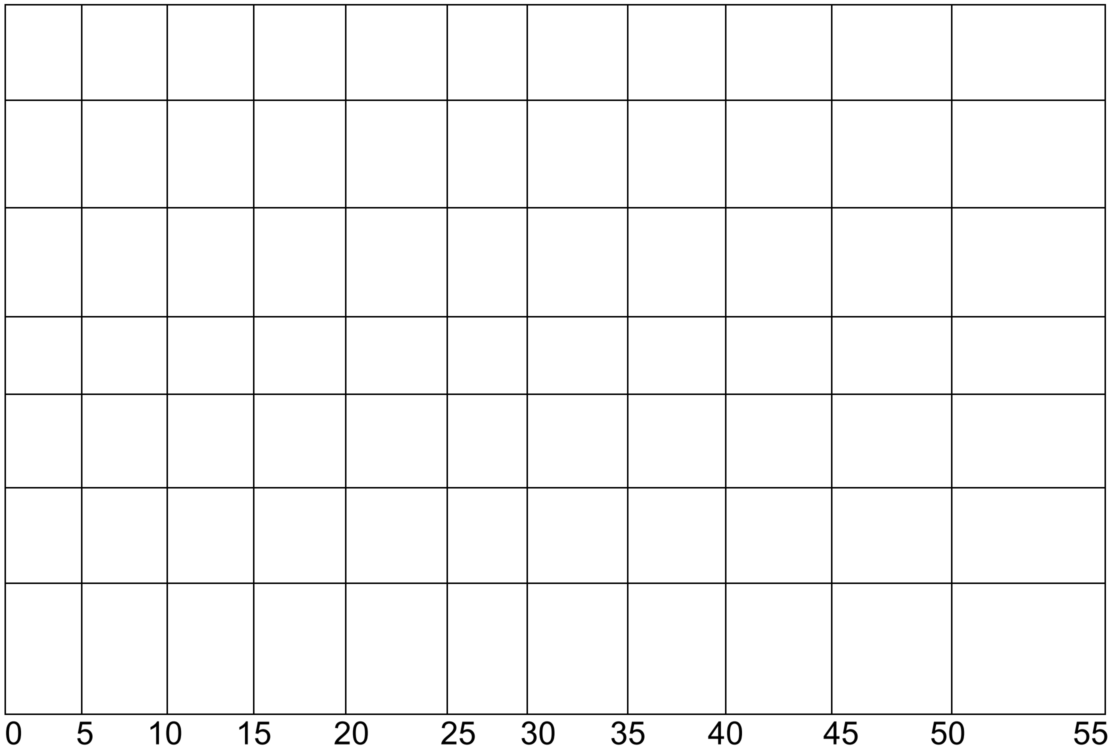
Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:
Super Bowl commercial 2014: Still increasing!

- The table below shows the cost of a 30-second Super Bowl TV advertisement each year since 1967. Create a scatter plot of the data on the grid below. When creating a scatter plot, do not connect the data points with lines as with a line graph. Be careful setting up the scale for the y-axis.
  1. How has the cost of a 30 second Super Bowl commercial grown over time? Describe this in detail. Has it increased or decreased? Has it grown at a constant rate of change?
  2. Try to draw a smooth curve that models the cost of an ad over time. It should go through many of the data points, but may not go through all of the data points. It should be a good model of how the cost is growing over time and be usable to make future predictions.
  3. As a group, try to find the quotient between each year of Super Bowl ads. Write these quotients to the right of the table on the first page. You should put the values to the right and between the two years that you computed. These values represent the number that we multiply by to get to the following year's cost of a Super Bowl ad.
  4. What does the typical quotient look like? Can you find an average? This average value represents a rough estimate of what we multiply by to get future ad costs.
- When we multiply by this value over and over again to find the next ad cost, we refer to it as the ***growth factor***. If you found that the typical quotient is 1.12, this means typically the Super Bowl cost increases by 12% per year and that our average growth factor is roughly 1.12.
- Use your typical growth factor to find the potential cost of a 30-second ad during the next three Super Bowls. For example, if your typical growth factor is 1.12, then multiply the last Super Bowl cost by 1.12 to find the potential cost of an ad during Super Bowl 48. Repeat this process until you have potential ad costs for Super Bowls 49 and 50.
- Graph your potential Super Bowl ad costs for these three Super Bowls on your graph. Do they seem to fit in with the trend?
- **Note to students:** You may notice that the cost of a Super Bowl ad has not grown linearly. The cost may have grown somewhat *exponentially.* This means that we can multiply by the same value year after year to find the cost of the following year's ad. To determine whether this is true, simply divide an ad cost by the cost from the year before.

NRS Level 6 Math
High Adult Secondary Education (Grade Levels 11.0 – 12.9)

Bowl number	Cost of 30-second ad
1	$40,000
2	$54,000
3	$67,500
4	$78,200
5	$72,000
6	$86,000
7	$103,500
8	$107,000
9	$110,000
10	$125,000
11	$162,000
12	$185,000
13	$222,000
14	$275,000
15	$324,300
16	$345,000
17	$400,000
18	$450,000
19	$500,000
20	$550,000
21	$575,000
22	$600,000
23	$675,000
24	$700,000
25	$800,000
26	$800,000
27	$850,000
28	$900,000
29	$1,000,000
30	$1,100,000
31	$1,200,000
32	$1,300,000
33	$1,600,000
34	$2,100,000
35	$2,050,000
36	$1,900,000
37	$2,100,000
38	$2,250,000
39	$2,400,000
40	$2,500,000
41	$2,600,000
42	$2,700,000



43	$3,000,000	
44	$2,800,000	
45	$3,100,000	
46	$3,500,000	

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Super Bowl Activity sheet
- Overhead projector
- Rulers
- Calculators

List of Technology Resources:

- Plato Learning Environment http://ple.platoweb.com/
- Purple Math www.purplemath.com

Building Functions (BF)

6.F.BF.1 / 6.F.BF.2 / 6.F.BF.3 / 6.F.BF.4 / 6.F.BF.5

Essential Understandings:

- Functions with a domain that are a subset of the integers and can be identified by the constant difference between consecutive terms (arithmetic sequences).
- Arithmetic sequences follow a discrete linear pattern, and the common difference is the slope of the line.
- Geometric sequences can be represented by both recursive and explicit formulas.
- Units, scales, data displays, and levels of accuracy are represented in real-world situations.
- Find, understand, and solve the inverse and composite relationship of functions.

NRS Level 6 Math
High Adult Secondary Education (Grade Levels 11.0 – 12.9)

Essential Questions:

- What is an arithmetic or geometric sequence and how does it relate to a function?
- What is the relationship between recursive and explicit equations and how are they represented symbolically?
- Which type of arithmetic or geometric sequence or function models a situation?
- How do you choose units, scale, data displays and levels of accuracy to appropriately represent a situation?
- How can the inverse and composite relationship of a function be used in a real-world situation?

Student will be able to:

- Use arithmetic operations to combine standard functions to create composite functions.
- Write arithmetic or geometric sequences both recursively or with an explicit formula to model a situation, translating between these two forms.
- Find the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(k(x)) and f(x + k) for specific values of a positive and negative k.
- Find the value of k given the graphs, explaining the effects on the graph using technology.
- Find inverse functions, solving an equation of the form f(x) = c for a simple function that has an inverse and rewriting an inverse expression.
- Verify the composition that one function is the inverse of another.
- Read values of an inverse function from a graph or a table, given that the function has an inverse.
- Create an invertible function from a non-invertible function by restricting the domain.
- Find the inverse relationship between an exponential and logarithmic function, using their inverse relationship to solve problems.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed practice problems from Khan Academy lessons on “Composing Functions” and “Manipulating Functions”
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals
- Evaluate level of group participation

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on composite functions using material from Khan Academy section "Composing Functions"
- Explain the concept and examples to students (partly through the videos) and ask pairs of students to work the sample and challenge problems and share answers for class review and discussion
- Present a lesson on finding the inverse of a function using material from Khan Academy unit "Manipulating Functions"
- Through a combination of whole-group teaching (including the videos) and small group work, have students complete all sections of the Functions unit: "Intro to Inverse Functions," the practice problems for the intro, "Inputs and Outputs of Inverse Functions," "Graphing the Inverse of a Linear Function," and the Practice section for evaluating inverse functions

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One–to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Overhead projector
- Notebooks
- Graph paper
- Graphing calculators
- Khan Academy material for "Composing Functions" and "Manipulating Functions"

List of Technology Resources:

- Khan Academy - https://www.khanacademy.org/math/algebra2/manipulating-

functions/funciton-composition/v/function-composition and https://www.khanacademy.org/math/algebra2/manipulating-functions/introduction-to-inverses-of-functions/v/introduction-to-function-inverses
- Composite functions - http://www.purplemath.com/modules/fcncomp3.htm

Linear, Quadratic, and Exponential Models (LE)
6.F.LE.1 / 6.F.LE.2 / 6.F.LE.3 / 6.F.LE.4

Essential Understandings:
- Differences between linear and exponential functions allow these functions to be used as an appropriate model.
- Use graphs and tables to recognize a situation where a constant grows or decays by a constant percent rate.
- Interpret the parameters in a linear or exponential function in terms of a real-world context (e.g., compounding returns or investment goals).

Essential Questions:
- What are the different ways an exponential or linear function can be compared?
- How can the parameters of a linear or exponential function be interpreted?
- How is it determined when a situation is best modeled by an exponential or linear function?

Student will be able to:
- State the differences between a linear and exponential function and determine which function best models a particular situation.
- Prove linear functions grow by an equal difference over equal intervals.
- Prove exponential functions grow by equal factors over equal intervals.
- Create graphs and tables to determine whether the constant grows or decays and by what constant percentage rate.
- Write a logarithmic equation for exponential models, evaluating the logarithm using technology.
- Interprets and states the parameters of either a linear or exponential function in terms of its real-world context.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:
- Complete and accurate solutions to practice problems from Khan Academy “Introduction to Exponential Functions” unit and Purple Math “Exponential Functions” unit

- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:
- Math journals
- Evaluate level of participation in group work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:
- Present a lesson on linear and exponential functions using material from Khan Academy section “Introduction to Exponential Functions”
- Explain the concept and examples to students (partly through the videos) and ask pairs of students to work the sample and challenge problems and share answers for class review and discussion
- Present a lesson on exponential functions using material from PurpleMath.com unit “Exponential Functions”
- Through a combination of whole-group teaching and small group work, have students complete all five sections of the Exponential functions unit

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)
- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:
- Graph paper
- Rulers
- PurpleMath.com unit on Exponential Functions (5 pages)
- Khan Academy videos and practice problems on Exponential Functions (2 videos, 2 sets of practice)

List of Technology Resources:

NRS Level 6 Math
High Adult Secondary Education (Grade Levels 11.0 – 12.9)

- Plato Learning Environment — http://ple.platoweb.com/
- Purple Math — http://www.purplemath.com/modules/expofcns.htm
- Khan Academy - https://www.khanacademy.org/math/algebra/introduction-to-exponential-functions/comparing-exponential-and-polynomial-functions/v/linear-exponential-models

Trigonometric Functions (TF)

6.F.TF.1 / 6.F.TF.2 / 6.F.TF.3 / 6.F.TF.4 / 6.F.TF.5 / 6.F.TF.6 / 6.F.TF.7 / 6.F.TF.8 / 6.F.TF.9

Essential Understandings:

- The unit circle allows all real numbers to work in trigonometric functions.
- Pythagorean identities can be proven and used to solve problems with specified context.
- Key features in a unit circle shed light on the relationships between two quantities.
- Trigonometric functions can be represented by a table, graph, verbal description or equation, and each representation can be transferred to another representation.
- Specific transformations occur to trigonometric functions based on a value *k* and its manipulation to the function.
- The trigonometric functions sin(x), cos(x), or tan(x) can be used to model real-life situations that exhibit periodic behavior.
- Changing parameters such as amplitude, period, and midline of a function will alter its graph and that these parameters are related to the context or phenomena being modeled.
- The trigonometric functions of sine, cosine, and tangent can be used to solve sum or difference problems.
- Using technology, evaluate and interpret inverse functions to solve trigonometric equations arising in real-world situations.
- Use the unit circle to explain symmetry with odd and even and periodicity of trigonometric functions.

Essential Questions:

- How can the unit circle be read and interpreted using radians?
- How does the Pythagorean theorem and the unit circle relate to the identity $\sin^2(x) + \cos^2(x) = 1$?
- What do the key features or characteristics of a trigonometric function represent?
- What are the different ways a trigonometric function can be represented?
- What transformations can occur to a trigonometric function/graph?
- How can the graphs of trigonometric functions be modified to best fit the situations being modeled?

- How do factors such as amplitude, period, midline, and horizontal shift affect these functions and relate to the phenomena being modeled?
- How can the trigonometric functions of sine, cosine, and tangent be used to solve sum or difference problems?
- How can technology be used to evaluate and interpret inverse functions to solve trigonometric equations arising in real-world situations?
- How can the unit circle explain symmetry with odd and even and periodicity of trigonometric functions?

Student will be able to:

- Use the radian measure of an angle as the length of the arc of the unit circle.
- Describe the unit circle and the basic trigonometric functions to shed light on the relationships between two quantities, interpreting the radian measure of an angle traverses counterclockwise around the unit circle.
- Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3,}$ $\frac{\pi}{4,}$ and $\frac{\pi}{6}$.
- Use the unit circle to express the values of sine, cosine, and tangent for $\pi - x, \pi + x,$
  and $2\pi - x$ in terms of their values for x, where x is any real number.
- Explain odd and even symmetry and periodicity of trigonometric functions using the unit circle.
- Write trigonometric functions that model periodic phenomena with specified amplitude, frequency, and midline.
- Restrict the domain of a trigonometric function on which the function is always increasing or always decreasing, allowing its inverse to be created.
- Uses inverse trigonometric functions to solve equations that arise in a modeling context, evaluating the solutions with or without technology and interpret the solutions in terms of a context.
- Prove the Pythagorean identities and use these identities to solve problems within a specific context.
- Use the sum and the difference of sine, cosine, and tangent to solve real world situations.

Evidence for Assessing Learning

Performance Tasks
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate solutions to problems from Purple Math lessons
- Plato Learning Environment tests-monitor management system by objective and NRS level

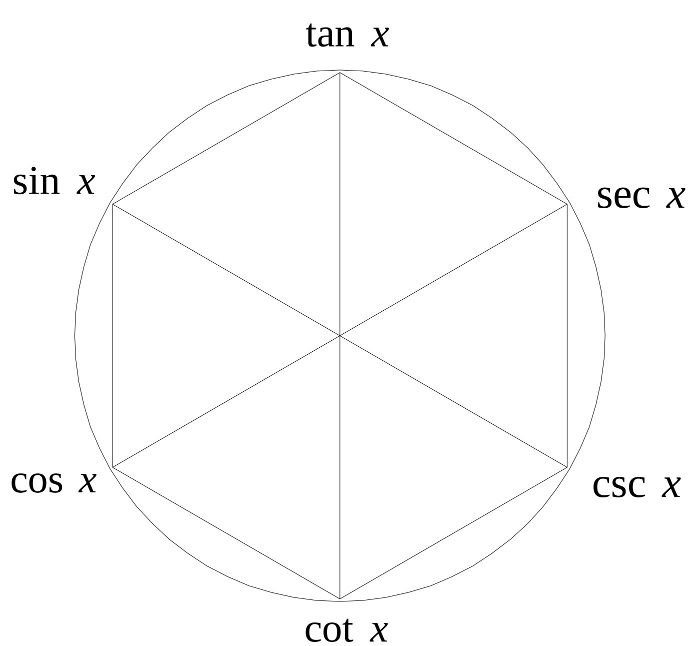
Other Evidence:

- Math journals
- Evaluate participation in group discussion and work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Provide an overview of trigonometric functions as shown below:

<table>
<tr><td colspan="2">The Magic Rule of Trigonometry: $\sin^2 x + \cos^2 x = 1$
Note: The Magic Rule is derived using a Unit Circle with an inscribed right triangle and the Pythagorean Theorem.
$(\sin x = \text{vertical distance})(\cos x = \text{horizontal distance})(\text{hypotenuse} = 1)$</td></tr>
<tr><td>Derivation #1 from the Magic Rule:
$Given: \quad \sin^2 x + \cos^2 x = 1$
$D\left(\sin^2 x\right) \quad \frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$
$Simplify \quad 1 + \cot^2 x = \csc^2 x$
$or \quad 1 = \csc^2 x - \cot^2 x$
$or \quad \csc^2 x - \cot^2 x = 1$</td><td>Derivation #2 from the Magic Rule:
$Given: \quad \sin^2 x + \cos^2 x = 1$
$D\left(\cos^2 x\right) \quad \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$
$Simplify \quad \tan^2 x + 1 = \sec^2 x$
$or \quad 1 = \sec^2 x - \tan^2 x$
$or \quad \sec^2 x - \tan^2 x = 1$</td></tr>
<tr><td colspan="2">Trigonometric Identity Hexagon

</td></tr>
<tr><td colspan="2">Understanding the Trigonometric Identity Hexagon:
1. The two trigonometric functions at the ends of any diameter are reciprocals of one another.</td></tr>
</table>

2. Every trigonometric function is the product of the trigonometric functions on either side of it. For example: $\sin x = \tan x \cos x$ and $\cos x = \sin x \cot x$
3. Each trigonometric function is equal to either of its adjacent trigonometric functions divided by its adjacent trigonometric function. For example: $\tan x = \frac{\sin x}{\cos x}$ and, also $\tan x = \frac{\sec x}{\csc x}$.
4. The product of any three non-adjacent functions is always 1. For example: $\tan x \cos x \csc x = 1$ and $\sin x \cot x \sec x = 1$.

NOTES: This mnemonic won't work if the Trigonometric Identity Hexagon is not drawn with the trigonometric functions at the correct vertices. A key for reproducing the mnemonic is to remember sin, tan, and sec for the three vertices at the top of the hexagon. Their co-functions are immediately below them. A total of 26 trigonometric identities can be obtained from this mnemonic. Have the students work in groups to derive and prove the 26 trigonometric identities depicted in the Trigonometric Identity Hexagon.

- Present a mini-lesson on Trigonometric Functions from Purple Math
- Discuss content and model sample problems and solutions as whole class
- Students independently study the "The Unit Circle" lesson from Purple Math and record a summary of the content in their math journals

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology– lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Scientific calculator
- Notebook
- PurpleMath.com unit on "Trigonometric Functions and Their Graphs" (3 pages)
- PurpleMath.com unit on "The Unit Circle"

List of Technology Resources:

- Purple Math lesson on Trigonometric Functions - http://www.purplemath.com/modules/triggrph.htm
- Purple Math lesson on The Unit Circle - http://www.purplemath.com/modules/unitcirc.htm

GEOMETRY (G)
Similarity, Right Triangles, and Trigonometry (SRT)
6.G.SRT.1 / 6.G.SRT.2 / G.SRT.3 / 6.G.SRT.4 / 6.G.SRT.5 / 6.G.SRT.6

Essential Understandings:
- The ratios of the sides of right triangles are functions of the acute angles of the triangle.
- The sine of an acute angle in a right triangle is equal to the cosine of that angle's complement (and vice versa).
- The Pythagorean Theorem applies only to right triangles.
- Derive the formula *A=1/2ab* sin (C) for the area of the triangle.
- Prove the Laws of Sine and Cosine for right triangle trigonometry in real-world situations.
- Prove the Laws of Sine and Cosine for non-right triangle trigonometry in real-world situations.

Essential Questions:
- How does similarity give rise to the trigonometric ratios?
- How do the trigonometric ratios of complementary angles relate to one another?
- How can the Pythagorean Theorem be used to solve problems involving triangles?
- How can the formula *A=1/2ab* sin (C) for the area of the triangle be derived and used?
- How can the Laws of Sine and Cosine for right triangle trigonometry in real-world situations be proved?
- How can the Laws of Sine and Cosine for non-right triangle trigonometry in real-world situations be proved?

Student will be able to:
- Use similarity to show that the side ratios in a right triangle are properties of the angles in the triangle.
- Use the definitions of trigonometric ratios for acute angle in a right triangle.
- Explain and use the relationship between sine and cosine of complementary angles.
- Apply and use trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems.

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- Derive an area of a triangle formula using the sine trig function.
- Prove and use the Law of Sine and Law of Cosine on both right triangle trigonometry and non-right triangle trigonometry in real world situations.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Completed trigonometric functions worksheet
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals
- Monitor group work and discussion of Khan Academy practice section

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plan

- Preset a lesson on trigonometric functions as ratios of the sides or a right triangle using material from https://www.khanacademy.org/math/geometry/hs-geo-trig
- Review the six trigonometric functions
- Students work as a class to solve the five problems under “Practice: Solve for a Side in Right Triangles” on Khan Academy
- Present the following worksheet for independent practice, applying trigonometry ratios of:

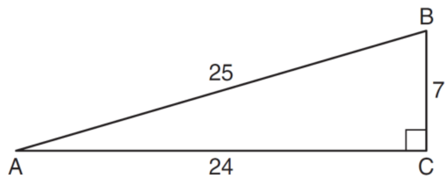
Sine= $\frac{\text{opposite}}{\text{hypotenuse}}$ Cosine = $\frac{\text{adjacent}}{\text{hypotenuse}}$ Tangent = $\frac{\text{opposite}}{\text{hypotenuse}}$

(The other three trig functions – cotangent, secant and cosecant are defined in terms of the first three. They are used less often, but they simplify some problems, as in the worksheet below)

Cotangent = 1/(tanA) Secant = 1/(cosA) Cosecant = 1/(sinA)

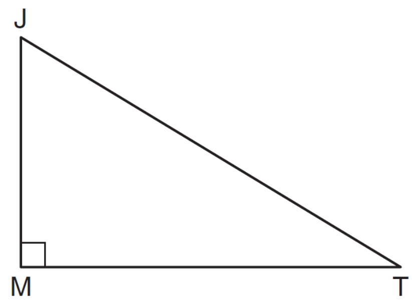
NRS Level 6 – High Adult Secondary Education (Grade Levels 11.0 – 12.9)

1 Which ratio represents $\csc A$ in the diagram below?



1) $\frac{25}{24}$
2) $\frac{25}{7}$
3) $\frac{24}{7}$
4) $\frac{7}{24}$

2 In the diagram below of right triangle *JTM*, $JT = 12$, $JM = 6$, and $m\angle JMT = 90$.



What is the value of $\cot J$?

1) $\frac{\sqrt{3}}{3}$
2) 2
3) $\sqrt{3}$
4) $\frac{2\sqrt{3}}{3}$

3 In the diagram below of right triangle *KTW*, $KW = 6$, $KT = 5$, and $m\angle KTW = 90$.

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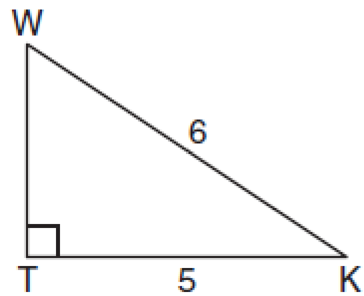
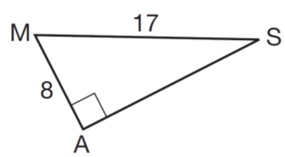


What is the measure of $\angle K$, to the *nearest minute*?

1) $33°33'$
2) $33°34'$
3) $33°55'$
4) $33°56'$

4 In the right triangle shown below, what is the measure of angle *S*, to the *nearest minute*?



1) $28°1'$
2) $28°4'$
3) $61°56'$
4) $61°93'$

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- https://www.khanacademy.org/math/geometry/hs-geo-trig#concept-intro

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Expressing Geometric Properties with Equations (GPE)

6.G.GPE.1 / 6.G.GPE.2 / 6.G.GPE.3 / 6.G.GPE.4

Essential Understandings:

- The equation of a circle can be found given a center and radius length.
- The equation of a parabola can be found given a focus and directrix.
- The equation of an ellipse and hyperbola can be found given the foci or the sum/difference of distances from the foci.
- Use coordinates to prove simple geometric theorems algebraically.

Essential Questions:

- How can the equation of a circle be found given a center and radius length?
- How can the equation of a parabola be found given a focus and directrix?
- How can the equation of an ellipse and hyperbola be found given the foci or the sum/difference of distances from the foci?
- Using coordinates, how can simple geometric theorems be proven algebraically?

Student will be able to:

- Write the equation of a circle given the circle's center and radius length.
- Write the equation of a parabola given the parabola's focus and directrix.
- Write the equation of an ellipse or hyperbola given the foci and/or the sum/difference of the distances of from the foci.
- Use the coordinates in a coordinate place to prove simple geometric theorems algebraically.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate notes and practice problems from Khan Academy
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Evaluate level of participation in group discussion and work

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on Conic Sections with material from Khan Academy PreCalc unit – Conic Sections
- Use the videos and challenge/practice problems to work through the content as a whole group.

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- Demonstrate how to derive an equation of a circle using the Pythagorean Theorem in the coordinate plane
- On an overhead projector or using smart room technology, model the following problem: A diagram of a circle with center at the origin and a radius with a length of 5 units is drawn in the coordinate plane. The points (5,0) (0,5), (-5,0), and (0,-5) are points on the circle. What other points are on the circle and what is the equation of the circle?
- Let P*(x,y)* be any other point on the circle. From P, draw a vertical line segment to the x-axis. Let this be point Q. Then triangle OPQ is a right triangle with OQ = x, PQ = y and OP = 5. We can then use the Pythagorean Theorem to write an equation of a circle: $OQ^2 + PQ^2 = OP^2$ or $x^2 + y^2 = 5^2$
- The points (3,4), (4,3), (-3,4), (-4,3), (-3,-4), -4,-3), (3,-4) and (4, -3) appear to be points on the circle and all make the equation $x^2 + y^2 = 5^2$ true. If we replace 5 by the length of any radius, r, the equation of a circle whose center is at the origin is: $x^2 + y^2 = r^2$
- Present a lesson on finding the equation of a parabola from Khan Academy (including the videos) and complete the practice problems as a whole class

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology – lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Overhead projector
- Graph paper and protractors
- Khan Academy unit on Conic Sections
- Khan Academy unit on "Focus and Directrix of a Parabola"

List of Technology Resources:

- Khan Academy "Conic Sections" - https://www.khanacademy.org/math/precalculus/conics-precalc
- Khan Academy "Focus and Directrix of a Parabola" - https://www.khanacademy.org/math/algebra2/intro-to-conics-alg2/focus-and-directrix-of-a-parabola-alg2/v/focus-and-directrix-introduction

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Geometric Measurement and Dimension (GMD)
6.G.GMD.1 / 6.G.GMD.2 / 6.G.GMD.3

Essential Understandings:

- Given an informal argument, explain the formulas for the circumference, area of a circle, volume of a cylinder, pyramid, and cone can be solved.
- Given an informal argument using Cavalieri's principle, explain the formulas of a sphere and other solid figures can be solved.
- Identify the shapes of two-dimensional cross-sections of three-dimensional objects.
- Identify three-dimensional objects generated by rotations of two-dimensional objects.

Essential Questions:

- How can an informal argument explain the formulas for the circumference, area of a circle, volume of a cylinder, pyramid, and cone?
- How can an informal argument using Cavalieri's principle explain the formulas of a sphere and other solid figures?
- How can shapes of two-dimensional cross-sections of three-dimensional objects be identified?
- How can shapes of three-dimensional objects generated by rotations of two-dimensional objects be identified?

Student will be able to:

- Use an informal argument to explain the formulas for the circumference and area of a circle, the volume of a cylinder, a pyramid, and cone.
- Use Cavalieri's principle to explain the formula of a sphere and other solid geometric figures.
- Identify and state the two-dimensional cross-section of a three-dimensional object.
- Identify the shapes of three-dimensional objects generated by rotations of two-dimensional objects.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Written explanations for formulas (by group)
- Complete and accurate solutions to Khan Academy practice problems (by group or independently)
- Plato Learning Environment tests-monitor management system by objective and NRS level

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Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on solid geometry using material from Khan Academy unit on Solid Geometry
- Working in groups students discuss and write in words explanations for the formulas for circumference and area of a circle, and volume of a cylinder, pyramid, and cone
- Groups present their explanations and class votes for the best for each formula
- As a whole class, present a lesson on 2D vs. 3D objects using videos and material from Khan Academy
- Students complete the practice sections in small groups or independently

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology – lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates.
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Khan Academy Solid Geometry lesson and practice
- Khan Academy 2D vs. 3D Objects lesson and practice

List of Technology Resources:

- Khan Academy – Solid Geometry
  https://www.khanacademy.org/math/geometry/hs-geo-solids/hs-geo-solids-intro/e/solid_geometry
- Khan Academy – 2D vs. 3D Objects
  https://www.khanacademy.org/math/geometry/hs-geo-solids/hs-geo-2d-vs-3d/e/cross-sections-of-3d-shapes

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Modeling with Geometry (MG)
6.G.MG.1

Essential Understanding:
- Geometric objects may be used to model various physical phenomena.

Essential Question:
- How can geometric figures be used to model physical phenomena or problem situations?

Student will be able to:
- Use and solve various geometric objects to model physical phenomena or problem situations.

Evidence for Assessing Learning
Performance Tasks:

Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:
- Evaluate and monitor group and independent work
- Completed design project for farmer's market

Other Evidence:
- Math journals

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:
- Option: open by showing a video from Inside Mathematics where a teacher helps student apply principles of quadrilaterals to the real-life problem of manufacturing kites. Use this as a prompt for discussion of other real-life application of math principles
- Explore design problems that exist in local communities, such as building a shed with maximum capacity in a small area or locating a dog park for three communities in a desirable area. Discuss challenges such as physical constraints and ways to minimize costs
- Propose a problem for groups to discuss: Maximize the number of vendors in a given space for a farmer's market along a narrow path of a lot. Work with given constraints such as standard stall sizes, distance required between stalls, etc. Create a design and justify the work with geometric methods

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Peer teaching through group work
- Interactive technology as assigned by instructor to support instruction such as: Plato Learning Technology – lessons by objective/ NRS level, interactive websites/illustrations, Study Stack – vocabulary support
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Additional practice with concepts and procedures in different contexts
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Picture of local farmer's market and estimate of physical statistics such as area, perimeter, etc.
- Notebooks

List of Technology Resources:

- Inside Mathematics – Video of lesson applying principles of quadrilaterals to real-life problem http://www.insidemathematics.org/common-core-resources/mathematical-practice-standards/standard-4-model-with-mathematics

STATISTICS (S)

Conditional Probability and the Rules of Probability (CP)

6.S.CP.1 / 6.C.SP.2 / 6.S.CP.3 / 6.S.CP.4 / 6.S.CP.5 / 6.S.CP.6 / 6.S.CP.7 / 6.S.CP.8 / 6.S.CP.9

Essential Understandings:

- Events can be described as a subset of a sample space.
- The probability of two events occurring together is the product of their probabilities, if and only if then the events are independent.
- The probability of two events can be conditional on each other and the interpretation of that probability.
- Two-way frequency tables can be used to decide if events are independent and to find conditional probabilities.
- Conditional probability and independence are applied to everyday situations.
- Conditional probability of A given B can be found and interpreted in context.
- The Addition or Multiplication Rule can be applied and the resulting probability can be interpreted in a context or in terms of a given model.
- Permutations and combinations can be used to compute probabilities of compound events in problem-solving situations.

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Essential Questions:

- How can an event be described as a subset of outcomes using correct set notation?
- How are probabilities, including joint probabilities, of independent events calculated?
- How are probabilities of independent events compared to their joint probability?
- How does conditional probability apply to real-life events?
- How are two-way frequency tables used to model real-life data?
- How are conditional probabilities and independence interpreted in relation to a situation?
- What is the difference between compound and conditional probabilities?
- How is the probability of event (A or B) found?
- How can the Addition or Multiplication Rule be applied and the resulting probability be interpreted within a context or in terms of a given model?
- How can permutations and combinations be used to compute probabilities of compound events in problem-solving situations?

Student will be able to:

- Describe events as a subset of a sample space, using characteristics of categories of the outcomes, or as unions, intersections, or complements of other events.
- Use the product of two probabilities when the events in a given situation are independent events, using this characterization to determine if they are independent.
- Create everyday situations that use conditional probability and independent events
- Find that the conditional probability of event A given event B and interpret this probability in the context given.
- Recognize, explain, and use the concepts of conditional probability and independence in everyday language and in everyday situations.
- Use the Addition or Multiplication Rule and interpret the appropriate rule in a context or in terms of a given model.
- Use permutations and combinations to compute probabilities of compound events in problem solving situation.
- Use and construct two-way frequency tables for independent events and find the conditional probability.

Evidence for Assessing Learning

Performance Tasks:
Demonstrate mastery of objectives through the assessment of graded: homework, worksheets, quantitative (numerically graded) rubrics, quizzes, tests, and other formal assessments. Including but not limited to:

- Complete and accurate problems in Steck Vaugn and New Readers Press text (if using as intervention or re-teaching option)
- Complete and accurate practice problems on Khan Academy (as group or independent work)
- Three complete and accurate worksheets from Explorations with Chance lesson (as group or independent work)
- Plato Learning Environment tests-monitor management system by objective and NRS level

Other Evidence:

- Math journals

Building the Learning Plan

Sample Classroom Activities and/or Lesson Plans:

- Present a lesson on conditional probability, beginning with material from Khan Academy Unit “Randomized Algorithms”
- Have students work in groups to complete the challenge questions and problems
- During the next session, present the “Explorations with Chance” lesson from NCTM Illuminations website. Students will explore conditional probability and how to compute probability of a compound event by playing and analyzing three different games of chance
- Student complete “Is It Fair?” worksheet, “A Fair Hopper” Activity Sheet, and the “Happy Hopper” Activity Sheet, either in groups or independently
- In their math journals students record their impressions of games of chance and the mathematical reasons for their opinions

Learning Activities:
(Interventions for students who are not progressing, instructional strategies, differentiated instruction, re-teaching options)

- Review concepts of probability in a textbook context (New Readers Press and Steck-Vaughn materials – see below)
- Peer teaching through group work
- One-to-one intervention
- Think out loud (demonstrate how to think about a problem)
- Provide students with a typed set of notes from their classmates
- Provide support around math specific and general vocabulary
- Universal Design for Learning protocols such as additional time, modified lesson for disabilities (i.e., enlarged print, drills, flashcards)

List of Instructional Materials:

- Handouts and lesson outline for Illuminations “Explorations with Chance” lesson
- Notebooks
- Coins

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- Chips with faces of red-red, red-blue, red-white, and white-blue
- *Mathematical Reasoning: GED® Preparation for the 2014 Test*. (2013). Steck-Vaughn. (Unit 2 Lesson 4)
- *Scoreboost 2014 - Graphs, Data, and Probability*. (2013). New Readers Press. (pages 26-37)

List of Technology Resources:

- Khan Academy www.khanacademy.org
- Purple Math www.purplemath.com
- NCTM Illuminations – Lesson on Explorations with Chance http://illuminations.nctm.org/Lesson.aspx?id=1145